UNIVERSITY OF PENNSYLVANIA

DIRECT METHOD OF DETERMINING CYCLICAL FLUCTUATIONS OF ECONOMIC DATA

MARTIN ALLEN BRUMBAUGH

A THESIS

IN STATISTICS

PRESENTED TO THE FACULTY OF THE GRADUATE SCHOOL IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

PHILADELPHIA

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FOREWORD

It is the author's privilege to acknowledge gratefully the invaluable criticisms and constant guidance of Professor Robert Riegel of the Wharton School Faculty. It is likewise a pleasure to record the author's indebtedness to Dr. Lincoln W. Hall for his assistance and never failing inspiration through the years. The author is permanently indebted to his colleague, Mr. J. P. Bursk, for the drawing of graphs and other assistance.

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I

INTRODUCTION

Scope of the Study.—A sharp distinction must be drawn between the economic problem of determining whether or not there exist any regularity in the so-called recurrent prosperous and depressed conditions of the business world, and the statistical problem of measuring such fluctuations on the assumption of their existence. There are two theories with which this book is not concerned. One is the relation of business cycles to economic theory, and the other is the problem of determining the cause of cyclical fluctuations. The vital nature of these theoretical considerations in connection with any study of cyclical fluctuations is fully recognized, but there is not space in, nor is it the ambition of, the present development either to add to or subtract from the existing literature on cycle theory.

It will perhaps be well to define more carefully what is involved in the assumption of the existence of cycles. We do not imply that there exists within business data a recurrent movement of fixed amplitude and period capable of expression in terms of elementary transcendental functions, but merely that the alternations of prosperity and depression have sufficient common properties to warrant quantitative expression and interpretation. This concept of the cycle follows naturally from ordinary experience in our dynamic civilization. For example, before amplitude was measurable "alternate prosperity and depression" were referred to by writers engaged in the study of business conditions.

The problem at hand can now be stated as an attempt to set up a method of measuring the cyclical movements existing in business data. It therefore becomes a purely statistical problem, and two methods of procedure are available. We may analyze

actual cases and draw conclusions from the analysis, or we may build up a method starting from certain hypotheses, and then apply it to actual cases to demonstrate its workableness. The latter method will be employed in this book.

CLASSIFICATION OF PRESENT METHODS OF MEASURING CYCLES.—Many methods of measuring business cycles are in use at the present time, and certain of these have similar properties whereas others differ fundamentally. They may for present purposes be classified on two bases; first, as to the thoroughness with which they segregate the several components comprising an economic time series, and secondly, as to the precision of measurement employed.

Under the first classification with respect to thoroughness of segregation we have:

1. Those which make no attempt to remove secular trend and seasonal variation prior to measuring the cycle.

2. Those which remove secular trend and seasonal variation, but include in the cycle any irregular changes due to unpredictable causes.

3. Those which remove secular trend and seasonal variation, and measure the cycle as a function with invariant amplitude and period, thereby excluding irregular changes.

Under the second classification with respect to precision of measurement we have:

1. Those which depend mainly upon graphs and free-hand methods, using only index numbers as a numerical aid.

2. Those which combine free-hand methods with accurate measurements, thereby producing a result which is partially independent of the person obtaining it.

3. Those which employ dispersion, correlated measures and algebraic formulas, thereby producing a result which is independent of the person obtaining it.

4. Those which express the regular components of a series in terms of mathematical equations, the application of which is purely formal in character.

Position of the Direct Method in These Classifications. —We are not interested here in group (1) of the first classification or either of the first two groups of the second classification. In dismissing these groups it is only fair to the authors of the methods included to state that they labored under the handicap of not being able to utilize the means of separating and measuring the components of a time series, first, because statistical technique was not sufficiently developed, and, secondly, because their results would have been incomprehensible to a majority of their readers. It is true, these methods are more readily understood by the "man on the street," but this is scarcely a pertinent argument for giving credence to them in statistical work, where the aim is scientific analysis of facts. To put it differently, to rely on these "rule of thumb" methods is to be satisfied with a first approximation long after more scientific formulation has been discovered.

The method to be developed presently, which for purposes of distinction will be called the "direct method," falls essentially in the second group of the first classification and in the third group of the second classification, being one which includes in the cycle irregular changes due to unpredictable causes, and which by rigid statistical methods is rendered independent of the persons employing it. No apology is offered for its position, although it is apparent that the last group of each classification is the maximum refinement. There seems, however, to be entirely too much regularity attributed to our economic cycle when it is assumed to possess regular periodicity, even though variable amplitude be obtained by inclusion of additional terms of a harmonic function. On these grounds the semi-empirical method of the middle groups has seemed preferable. Further, the concept of a cycle ought to be such as to accord to the greatest possible degree with actual business experience, and this would seem to be a condition of irregularity within regularity.

II

METHODS OF MEASURING CYCLES

A. Analysis of Methods Now in Use

Enumeration of Methods Included.—The methods now to be analyzed belong to the group which segregates secular trend, seasonal variation and cyclical fluctuation. There are individual differences between the methods, but their common characteristics are well known, so well known in fact that it seems unnecessary to reproduce the details of method of each. Suffice it to say that such methods as Person's "Link Relative,"[1] Harts' "Monthly Means,"[2] Hall's "Relative Monthly Means,"[3] Hall's "Moving Secular Trend and Moving Integration,"[4] Macaulay's "Moving Average,"[5] King's "Moving Median,"[6] etc., all included in the designation "older methods" used hereafter in this book, are being described.

Form of Analysis.—The components of a series of data may be expressed in the form of an equation thus,

$$O = T + S + C$$

in which T, S and C represent trend, seasonal variation and cyclical fluctuation, respectively. The goal of the analysis of such a series is usually measurement of the cyclical fluctuations. The methods under discussion arrive at this goal by successively removing the other two components. The usual order is, first,

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[1] *Harvard Review of Economic Statistics*, January, 1919, pp. 8–37.

[2] *Journal of the American Statistical Association*, September, 1922, Vol. 17, No. 139, pp. 341–349.

[3] *Journal of the American Statistical Association*, June, 1924, Vol. 19, No. 146, pp. 156–166.

[4] *Journal of the American Statistical Association*, March, 1925, Vol. 20, No. 149, pp. 13–24.

[5] *Federal Reserve Bulletin*, December, 1922, pp. 1414–1421.

[6] *Journal of the American Statistical Association*, September, 1924, Vol. 19, No. 147, pp. 301–313.

to measure and remove the trend by subtracting its influence item by item and the equation becomes,

$$O - T = S + C$$

These residuals, O − T, are really a first approximation to the cyclical values.

The second step is to segregate the seasonal variation and remove it. This is likewise accomplished by subtracting its influence item by item from the residuals, O − T, and the status of the equation is,

$$(O - T) - S = C$$

A third step reduces the results to relatives of the trend and we have finally,

$$\frac{C}{T} = \text{Relative Cycles}$$

This process may be summarized by a quotation from Persons:[7] "The method adopted of correcting the original data for secular trend and seasonal variation was as follows: (a) the monthly ordinates of the line of secular trend were computed; (b) these ordinates were multiplied by the indices of seasonal variation; (c) the resulting products were subtracted from the corresponding items of the original series; (d) the resulting deviations, plus or minus, were expressed as percentages of the secular trend."

Nature of Trend.—It will be necessary to understand a little more clearly the nature of the trend which is being removed by the method of the preceding section. We seem to have passed through a sort of cycle of methods of representing trend. Several years ago a moving average with a reasonably large number of years as base was used. Then came the popularity of the straight line as a representation of trend. We seem now to be on the verge of abandoning the straight line, because there are secular movements—changes of a fundamental nature—occurring in data over a period of years. These preclude representation by a single straight line—for example, the changes in our financial and banking data resulting from the introduction

[7] *Harvard Review of Economic Statistics*, January, 1919, p. 35.

of the Federal Reserve System, or the shifts in consumptive demand resulting from changed social conditions of the post-war period. The indication, then, is that something other than a straight line will be used to measure trend in the future. The question of what shall be used is particularly significant because a majority of the methods employ trend ordinates as a base for expressing cyclical fluctuations in relative form.

The growth component of different series of data will take different forms. For instance, the growth of populations is properly represented by a curve of increasing slope because such growth tends to be in geometric progression. But it is not possible to formulate any general laws of growth for economic data, because so many interrelated forces are operative simultaneously that the net resultant of their combined action is not predictable. For this reason we are under the necessity of making an assumption concerning what portion of any series shall be considered as growth or trend. This assumption must lead to reasonable results in practice, if it is to be of any value. Consequently, the concept should be as flexible as possible, but it will necessarily have to be definite enough to insure the removal of trend from a series which is being analyzed.

If we assume that trend is properly represented by a straight line, we immediately eliminate all possibility of changes in the rate of growth of the data, likewise we eliminate the possibility of trend absorbing anything from the short-time oscillations which are superimposed upon it. This seems to be a quite artificial concept, and in a great many cases results in a highly unrepresentative trend. This unrepresentative character of straight line trend is a double source of error, (1) in its removal, (2) in its use as the base for relative cycles. There are two avenues of escape from this dilemma; one is to determine the kind of trend which shall be used for each set of data at the time the analysis is made, the other is to make an assumption concerning trend which is sufficiently elastic to be applicable to all cases. The latter escape is decidedly preferable, because the former is not independent of the manipulator.

Nature of Seasonal Variation.—The usual assumption concerning seasonal variation is that it repeats itself with equal

intensity, either absolute or relative, at the same seasons from year to year. No one has held at any time that these short-time movements will actually obey any such regular law as the assumption ascribes to them. There is, however, evidence from actual experience of the existence of considerable regularity in the seasonal variations.

This assumption will be discussed from two points of view, the facility with which it describes the concept of seasonal variation and its ability to adjust itself to changes in the nature of the seasonal movements. If the portion of the data attributed to seasonal variation remains constant from year to year then differences occurring at the same seasons of different years are all attributed to the cycle. In general such differences are really of a cyclical nature, and consequently they are properly allocated, but particularly in series of data in which the seasonal changes are directly dependent upon the weather there will be differences at the same seasons of different years which are in no sense a result of business conditions. A case in point would be the monthly mileage of concrete highway constructed by a State Department before the development of frost-proof concrete mixing, and similarly for construction of concrete buildings. Another case would be monthly shipments of perishable provisions in northern climates. These are not cyclical movements yet they are placed with the cycle if we assume seasonal variation to be constant. It appears to the writer that a more elastic assumption concerning the removal of seasonal variation is needed.

There is a further cause for objection arising from the method of measuring and removing the seasonal variation. All of the methods under discussion compute seasonal movements by a process of averaging the data either in absolute or relative form for the several seasons over the period covered by the data.[8] The point about to be made applies to most of the methods in use, but can be most readily understood in connection with

[8] A partial exception occurs in King's *Moving Median Method.* He computes an average seasonal index, but distributes it before applying it. The objection to the method arises from the fact that the seasonal variation for a recent month or quarter cannot be computed but must be assumed.

Harts' "Method of Monthly Means." In this we obtain the seasonal variation by taking an arithmetic average of the residuals, O − T, for each season of the year over the period covered by the data. Suppose a case in which the seasonal variation for January is positive, but the data contains only cycles of three years' duration and by accident January always falls in the negative phase two of the three years; further let one of the Januaries of the negative phase always balance the January of the positive phase. If the average of these January items be taken the result will evidently be negative, although we assumed that the January seasonal variation should be positive. Of course, the answer to this objection to the method is apparent. In an actual case one has no knowledge of the seasonal variation prior to measuring it, hence the case given could not arise. But the conditions of the example above are not impossible, hence to say that they do not exist because the assumption of equal seasonal variation will not express them scarcely proves the validity of that assumption.

Changes in the nature of the seasonal variation present another problem. Suppose a case in which the amount of seasonal variation is undergoing a gradual reduction from year to year. Take, for example, sales of an ice-cream manufacturing plant. It would not be necessary to go back more than two decades to find winter sales of such a plant negligible. Ice cream belonged to the luxury class of commodities, and had, in particular, a summer demand. As such, a large part of sales volume could rightly be attributed to seasonal differences. But more recently ice-cream has been largely removed from the luxury class into the class of necessities. As a result its seasonal movements are entirely changed from what they were at the earlier time. This case is offered as an example of permanent or at least relatively permanent change in the nature of the seasonal variation. This situation is apparently not contemplated in the assumption of equal seasonal variation from year to year. In the case given seasonal values obtained by the averaging process would be too small early in the period covered and too large for the later items.

We may conclude that, insofar as the assumption of equal

seasonal variation from year to year does not fit the facts of any set of data, because of either short-time or long-time changes in the character of the seasonal component, the same situation arises which we encountered in the discussion of trend. The assumption is too inelastic to provide adequately for the description of the data, and the escape from the difficulty lies in establishing a more elastic assumption for the removal of the seasonal influence.

Inaccuracies Resulting From the Use of These Methods.—We have already pointed out the fact that a trend line which fails to fit a set of data introduces a double error into the computation of its relative cycles, that the removal of an unrepresentative seasonal variation is a further source of error. Taken altogether it appears that there is a fair probability of considerable inaccuracy in the cyclical remainders of present methods. This probable inaccuracy of results plus the excessive labor of performing the various operations requisite to measurement and removal of the trend and seasonal components, have brought up the question of whether it would be possible to substitute a more flexible set of assumptions for the present inelastic ones, in order to obtain a shorter solution of any problem and more accurate results. Such a set will be presented in the next section.

B. The Direct Method

The Problem.—We turn now to the description of a method of measuring cyclical fluctuations which will dispense with the preliminary steps of making inelastic assumptions concerning trend and seasonal variation, and then computing and removing the effects of each. In other words, this discussion is offered as an answer to the question, "Can cyclical fluctuations be measured directly?" Or stated at greater length and more explicitly the problem is, to set up flexible assumptions concerning the nature of secular trend and seasonal variation, such assumptions providing a sufficient description of these movements to permit of correction for them without measuring their amounts, thereby affording a *direct* measure of the cyclical component of a time

series, instead of reaching it as the last step of a long succession of operations. This statement is intended to emphasize the fact that no attempt will be made to segregate and measure either the trend or the seasonal variation. The sole purpose will be to arrive *directly* at a figure free from the influence of both.

In the development of the direct method two primary purposes have been kept constantly in mind:

1. To simplify the process of computing the cyclical fluctuations.

2. To broaden the assumptions concerning trend and seasonal variation in order to eliminate sources of error present in the older methods.

Both of these goals have been attained by alterations in the concepts of trend and seasonal variation, respectively, which permit their removal without measuring them in detail.

The inaccuracies of the older methods were attributed in the last section to the inelastic nature of both trend and seasonal variation as defined, but these definitions or their equivalent are essential to the methods in which they are employed. The totally different definitions of these components in the new method arise from the fact that it proceeds by different steps to the measurement of the cyclical fluctuation. But, before passing to the explanation of the direct method, it will be advisable to state the assumptions on which it is based.

Statement of the Assumptions.—The primary requisite of trend is that it shall fit the data to which it is applied both in a general sense, and in particular that it shall reflect the tendency of the short-time movements of the data. Hence, *we assume trend to be a mildly waving curve fitting the data and responding remotely to the short-time oscillations thereof.*

We desire a measure of seasonal variation which will express the tendency of the seasonal movements to repeat themselves from year to year, but which is elastic enough to adjust itself to changes in the nature of such movements. Hence, *we assume that the amount of any time series which is to be attributed to seasonal*

variation will not be materially different for the same seasons of any two adjoining years.[9]

Formulation of the Direct Method.—The method which is to be described naturally divides itself into two steps: (1) the measurement of cyclical changes exclusive of seasonal influence; and (2) a correction for what small residue of trend may remain, if any.

Elimination of Seasonal.—We can now explain in detail the first step of the method—direct ascertainment of the cyclical movements exclusive of seasonal influence. Consider any item of data say the ith month of the jth year of a given series and call it O_{ij}. The same month of the following year will be designated as O_{ij+1}. In general these items contain three components, trend, seasonal variation and cycle.[10]
That is,

(a) $$O_{ij} = T_{ij} + S_{ij} + C_{ij}$$

and the item for the same month of the next year would be

(b) $$O_{ij+1} = T_{ij+1} + S_{ij+1} + C_{ij+1}$$

Now we can say

(c) $$T_{ij+1} = T_{ij} + k_j$$

that is, the difference between the trend values at the same periods of two adjacent years will differ by an amount k_j which is constant if trend means straight-line trend and will have rather small variations in adjoining years whatever the definition of trend which is employed. And by the seasonal assumption,

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[9] In his development of *The Method of Monthly Means*, Hart's statemen of the seasonal variation assumption is: "The essential feature of a strictly seasonal variation is that its numerical value is the same, at corresponding months, in all years." Both the *Link Relative Method* of Persons and *The Method of Relative Monthly Means* of Hall, make the same assumption except for the word "numerical" which is changed to "relative."

[10] The reader will probably feel that his question as to just what is included in each component ought to be answered. The trend is assumed to be a non-periodic component either increasing or decreasing over any period under consideration. The cycle is assumed to be a periodic component with varying period and has included with it all variations in the data due to unpredictable causes. Seasonal variation is a component with period one year and capable of changing its amplitude over long periods only.

that is "the seasonal variation will not be materially different for the same seasons of adjoining years."

(d) $$S_{ij} = S_{ij+1}$$

hence dividing

$$\frac{O_{ij+1}}{O_{ij}}$$

we have after substituting (c) and (d)

$$\frac{T_{ij} + S_{ij} + C_{ij+1} + k_j}{T_{ij} + S_{ij} + C_{ij}}$$

Inspection of the numerator and denominator above shows that, expressed as index numbers, variations from 100 show cyclical changes. When C_{ij+1} equals C_i the trend residue (k_j) being ignored for the present, the result is 100. To the extent that C_{ij+1} exceeds C_{ij} the result exceeds 100. We will now see in what form these cyclical changes are expressed.

Let $\frac{O_{ij+1}}{O_{ij}} = 1+D$ where D is either positive or negative.

Then by the division property of proportion,

$$\frac{O_{ij+1} - O_{ij}}{O_{ij}} = D$$

that is, after substituting (c) and (d) above

$$(T_{ij} - T_{ij}) + (S_{ij} - S_{ij}) + (C_{ij+1} - C_{ij}) + k_j = D \cdot O_{ij}$$

So that,

(e) $$(C_{ij+1} - C_{ij}) + k_j = D \cdot O_{ij}$$

This expression reads: except for a yearly trend increment, k_j[11] the product of any item of data times the variation from 100 of its ratio to its successor in the following year, equals the absolute cyclical difference of the two items, that is, using the symbols employed above,

$$O_{ij} \cdot \left(\frac{O_{ij+1}}{O_{ij}} - 100\right) = C_{ij+1} - C_{ij}$$

[11] Note that k_j is not cumulative. It is simply the trend increment or decrement of one year's time regardless of the value of i and j, and consequently creates less difficulty than that encountered by the older methods which are forced to deal with trend as a cumulative growth component.

This gives us the basis of the method—divide each item into the item for the same season of the following year.[12] This gives a relative on the early year as a base whose variation from 100 is, except for a trend residue, the difference between the cyclical fluctuations of the two items. These indices contain no element of seasonal variation, nor do they contain residues resulting from changes in the nature of the seasonal variation, for any increase or decrease which becomes a permanent part of the data for a series of years will be eliminated from D and become a part of the base 100 when the divisions are performed.

To sum up, we have by the first step obtained a set of relatives each on its own base and each containing exclusively cycle and a trend residue. This trend residue is very small because there is in these relatives no growth factor affecting the items successively. All continuous growth is included in the 100, and only a single year's difference appears in D. There is, further, no seasonal variation left in D, all of it being embodied in the 100.

Correction For Trend Residue.—The next step will be the removal of the trend residue k_j. It will, perhaps be advisable to use an example to give an idea of k_j.

Suppose

i	j	O		T		S		C
Jan.	1923	1000	=	500	+	100	+	400
Jan.	1924	800	=	550	+	100	+	150

Then

$$\frac{\text{Jan. 1924}}{\text{Jan. 1923}} = \frac{800}{1000} = 80, \text{ and } D = -20$$

Now we had (e)

$$(C_{ij+1} - C_{ij}) + k_j = O_{ij} \cdot D_{ij+1}$$

substituting we have

$$(150 - 400) + (550 - 500) = 1000 \cdot -20$$

or

$$-250 + 50 = -200$$

The difference between the cycles, − 250, is the figure which is

[12] Thus in monthly data January of 1924 will be compared with January of 1923, February of 1924 with February of 1923, etc.

ultimately wanted so that the problem to be solved is how to eliminate from the relative variate D the influence of the trend residue, 50.

It was stated earlier that the trend was to be considered as a non-periodic function.[13] We can now define trend more carefully. It is scarcely likely that the growth of a set of data is properly represented by anything as regular as a straight line. There ought to be accelerations and retardations in the slope of the trend line operative indirectly from the business cycle or from any irregular movements. What effect any one or any group of these short-time factors may have upon the slope of the trend line is not accurately ascertainable, but any response which the trend might make would surely be in the direction of the short-time movement which induced it. It seems, then, that instead of assuming that trend absorbs nothing from the short-time movements which are superimposed upon it, a more representative notion of trend can be obtained from viewing it as the net resultant in the long run of the several forces combining to make up the total volume of any set of data in a short time. And just as reaction varies with stimulus in the nervous system, so trend should vary in harmony with the varying intensity of the short-time oscillations which are combined with it. This conception of trend may appear to involve complications not encountered when it is assumed to be a straight line. If such be the case, it is mainly because the assumption of a straight line to represent trend introduces into the notion a simplicity which it does not possess. As long as we are under the necessity of placing the exact nature of trend in the category of assumptions, for the purpose of specifically measuring and removing it, this oversimplification is desirable in the interest of a workable solution to the problem. In the present development we set out to remove the influence of trend without measuring it in detail. On that basis it is possible to broaden greatly the concept of trend by setting up a much more flexible assumption concerning its nature. Hence, as stated earlier, *trend will be considered as a slightly waving curve, fitting the data, and responding remotely to the short-time oscillations thereof.*

[13] See definition in footnote 2, page 16.

Perhaps the reader will feel that this is an altogether unsatisfactory definition of trend. If the trend must be measured ordinate by ordinate and removed, then a closer definition must be had, but it is a virtue of the present method that it requires no attempt to measure trend in detail. Only the trend residue k_j remaining after the ratios of items described earlier have been computed, requires adjustment.

Suppose we consider a series of data which contains only seasonal and cyclical movements. In that case, in the previous equation, (e),

$$(C_{ij+1} - C_{ij}) + k_j = D \cdot O_{ij},$$

the term k_j would be absent and the difference of the cycles would be expressed in relative form by $1 + D$. Now we are assuming that cyclical movements are irregularly periodic, consequently in the absence of trend the arithmetic average value of $1 + D$ for any number of complete cycles ought to be approximately 100. In face of the elastic assumption made concerning the nature of trend, it seems to be misplaced attention to attempt to trace the negligible amount by which this average would vary from 100. As a matter of fact the *geometric* average of the $1 + D$'s would equal 100, because, if the results gotten in the original data by dividing the early year by the recent year, be compared with the reverse method actually followed, it will be found that the results of one set are the reciprocals of the other set. If the corresponding pairs of relatives be multiplied together and the square root extracted, the result will be 100 in each case. Likewise if the n pairs of items of both direct and inverse be multiplied together and the $2nth$ root taken, the result will be 100. Hence the product of each set taken individually with the proper root will also be 100, provided always that only regularly periodic functions are affecting the result. In spite of this property of the geometric average, the increased labor of its computation does not seem to be compensated for by an equivalent gain in accuracy from its use, consequently we will adhere to the approximate assumption that so far as seasonal and cyclical movements are concerned the arithmetic average of the $1 + D$'s will be 100. We conclude, then, that in

a series containing trend, *i.e.*, a series in which k_j is present, the amount by which the average 1 + D varies from 100 expresses the disturbance due to k_j. But this variation, call it h, does not apply equally to all the 1 + D's because the absolute amount k_j is not changing appreciably from year to year, whereas the absolute amount of the data to which it applies is changing in a trend sense. *So in order to equalize the amounts to be added to or subtracted from the several 1 + D's we first multiply h by the ratio of the arithmetic average of the original data to each of the items and then apply the result to 1 + D.* The first question which naturally arises is as to the nature of the correction made by this process. Only an abstract answer will be attempted here, because a concrete answer is given with greater ease and clarity in the next chapter in the presence of actual data. By taking the ratio of the average of the items to each of the items we increase the relative amount attributed to k_j for all items less than the average, and decrease it for items greater than the average. The total correction for all items, however, is equal to the total variation of the 1 + D's. Also, by varying the relative amounts applied to the several items in inverse proportion to their relations to the average, we correct for k_j in fairly stable absolute amounts from year to year. This means that we have abandoned straight-line trend, but that the nature of the new trend is, according to the description presented a few pages back, namely "a slightly waving curve, fitting the data, and responding remotely to the short-time oscillations thereof."

The Form of the Result.—We have now accomplished in two major steps our initial purpose—to express cyclical fluctuations without previously measuring and correcting for the trend and seasonal movements. Thus the possible chance of error arising from the method of removing these components, as well as the laborious details of their measurement have been avoided. The present status of the indicator equation (e) is,

$(C_{ij+1} - C_{ij}) = O_{ij} \cdot (D - h')$ where h' is the correction for trend residue.

That is, the difference between the cyclical amounts for the same season of two successive years is expressed as a relative of the

original data of the earlier year. Let us examine this condition more closely.

$$\frac{C_{ij+1} - C_{ij}}{O_{ij}} = (D - h')$$

Suppose $C_{ij+1} = C_{ij}$, *i.e.*, no cyclical change present; then the value of $(D - h')$ would be 0 and the index would be 100 which is exactly what it should be. Now this condition may arise in two ways,

(f) $C_{ij+1} \equiv C_{ij} \equiv 0$, or (g) $C_{ij+1} = C_{ij} \neq 0$,

consequently, the relative $(D - h')$ must be understood to be an expression of *change* of cycle and not an expression for the *amount* of cycle.

Consistency of the Results.—We now have before us the philosophy of the method. Suppose two items of data for the same seasons of successive years contained no component of cycle, then after the trend residue had been eliminated the index gotten by taking their ratio would be 100. This 100 is interpreted not necessarily as the non-existence of a cyclical amount but as no change in cyclical amount. Let the items for the next month of the same years be expressed as a ratio. If after the trend residue has been removed this relative is less than 100, it shows, that cycle is changing to negative from positive. This affords the connecting link between the previous steps in which items were compared for the same seasons of adjoining years and the final interpretation of results with respect to consistency from month to month. We are dealing with, (1) rates of change and, (2) directions of change of the cyclical components, and these movements are continuous from month to month of the same year, although the derived relatives are based on the item for the same season of the preceding year. Hence the relatives $1 + (D - h')$can be taken as indices of cyclical change comparable with cyclical indices of other methods.

We must next consider more carefully the interpretation of the indices $1 + (D - h')$. We have already stated the condition required for an index of 100, − cycles equal for the two items

in question. Taking this ratio as a starting point let us see what may happen in the following month.

As notation we will take,

O_{i1} = the data for the first month of the first year.
O_{j1} = the data for the second month of the first year.
O_{i2} = the data for the first month of the second year.
O_{j2} = the data for the second month of the second year.

$$\frac{O_{i2}}{O_{i1}} = 1 + (D - h')_{i2} = (1 + H)_{i2}$$

$$\frac{O_{j2}}{O_{j1}} = 1 + (D - h')_{j2} = (1 + H)_{j2}$$

(We will assume in every case that trend residue has been corrected, hence let $D - h' = H$).

Attention is now directed to the relationships of the cyclical components of the last two equations above.

(1) If $C_{j2} > C_{i2}$ and $C_{j1} = C_{i1}$ then $(1 + H)_{j2} > 1$[14]

(2) If $C_{j2} < C_{i2}$ and $C_{j1} = C_{i1}$ then $(1 + H)_{j2} < 1$

These two cases are easy; they simply mean that with no change of cycle in the base year the relative $(1 + H)$ will follow the change of cycle in the new year.

(3) If $C_{j2} = C_{i2}$ and $C_{j1} > C_{i1}$ then $(1 + H)_{j2} < 1$

(4) If $C_{j2} = C_{i2}$ and $C_{j1} < C_{i1}$ then $(1 + H)_{j2} > 1$

It would seem that the results here are paradoxical, that a change has been indicated where none existed, because we assumed that no change in cycle occurred in the second year from the first month to the second month. But, if we keep in mind that we are not expressing merely the cyclical amount of the second year but the change between the first and second years, it is apparent that in (3) there has been a negative change in the second year, and in (4) a positive change.

(5) If $C_{j2} > C_{i2}$ and $C_{j1} > C_{i1}$, but $(C_{j2} - C_{i2}) < (C_{j1} - C_{i1})$ then $(1 + H)_{j2} < 1$

(6) If $C_{j2} < C_{i2}$ and $C_{j1} < C_{i1}$, but $(C_{j2} - C_{i2}) < (C_{j1} - C_{i1})$ then $(1 + H)_{j2} > 1$

[14] Or expressed as index numbers the values of $(1 + H)$ are greater than or less than 100 as the case may be.

These two conditions present the real difficulty of interpretation. In (5) the cycle for both years is increasing in absolute amount but it is increasing faster in the early year than it is in the recent year, consequently it might seem that a positive index should result. On the contrary, the negative index shows that the change is decreasing, and such a condition is prior to a real negative cyclical swing, or it is a condition on a positive swing which could not possibly continue for more than a month or two. In either case (5) is a diminishing of existing tendency and is of no more than secondary importance to the theory. What has been said concerning (5) is equally true in reverse order of (6) and its importance is also minor. The chance of either (5) or (6) occurring is rather small. It might almost be said that they are impossible unless the cycle is totally irregular. There seems to be a possibility that the appearance of (5) and (6) with any degree of regularity might be taken as a criterion of the irregularity of the cyclical movements of the data in which they appear. The reason for making this statement is the fact that any persistent variation in the same direction at the same season of adjoining years is by definition seasonal and therefore not operative. A movement such as this is not cyclical and consequently cannot persistently determine H.

We must next meet the argument that these indices $(1 + H)$ are not comparable since they are referred to such non-homogeneous bases. For instance this case might arise,

	T	S	C	O
1st year				
O_{i1} =	500 +	100 +	200 =	800
O_{j1} =	550 +	50 −	200 =	400
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
2nd year				
O_{i2} =	1050 +	100 −	150 =	1000
O_{j2} =	1100 +	50 −	650 =	500

In each case $(1 + D) = 125$, but in one case

$$D \cdot 800 = (-150 - 200) + k_j$$

while in the other

$$D \cdot 400 = (-650 + 200) + k_j$$

In each case $k_j = 550$, but when the average amount due to trend is obtained for an entire set of items and this trend is corrected by the ratio of the arithmetic average to these two items, 800 and 400, the final $(1 + H)$ will be different just as the cycle differences are different. The point to be made is that although the base to which the indices $(1 + H)$ are referred is non-homogeneous, its non-homogeneity is due to cycle changes and hence has its effect in both the numerator and denominator of $(1 + H)$. This is not a perfectly compensating relationship but it largely eliminates the criticism that non-homogeneity of base voids the continuity of the indices $(1 + H)$.

Comparison Between the Direct and Older Methods.— The final comparison to be made is between the cyclical differences of the new method and the cycles of the older methods. Using for the purpose the methods of "Link Relatives" and "Relative Monthly Means" because they are representative of two different types of cycle computation, but use similar modes of presenting results, we find both express the amount attributed to cycle for any period as a relative of trend. In both cases trend means a straight line or some non-periodic variation thereof. This is intended to give a regular base for fluctuations and since trend is a continuous function it affords continuous variation of relative cycles. To this concept no exception is taken. It is particularly easy of comprehension when straight-line trend is meant. But, suppose a case in which the type of trend incorrectly used was not the actual trend existing in the data (we stated earlier that it was extremely doubtful whether the actual amount of trend of a set of data ever could be known) then, by so much as the problematical trend used varies from the actual trend its use as a denominator for relative cycles is a purely artificial device. One might insist in reply that such variations are negligible, and with that view the writer thoroughly agrees. If a method is to be used which measures cycles as variations from a predetermined base, then it may be granted that the base ought to be the trend ascribed to the data, and not some other figure. The understandableness of the concept in its present form far outweighs any considerations of increased accuracy tending to lead to greater complexity.

If the present method is to be of any value in cyclical analysis, it must depart from the older methods at this point. Is it necessary to think of cycles in terms of their relation to trend? As long as trend was being more definitely defined, for example as a straight line, its use as a base for cycles could scarcely be disputed. But, we are casting considerable doubt in the present development upon the legitimacy of considering trend as inelastically defined. Since in the present direct method trend is very elastically defined or empirical in character, there are two very good reasons why it is not available as a base for cyclical fluctuations—first, we do not want as a base an imperfectly defined component, and secondly, we have not anywhere segregated trend, consequently it is not available for use. Then, what shall be used as a base? That which is used in the new method is the original data for the same season of the preceding year.

There is one other major difference between the direct method and older ones. The direct does not give at any time the value of an individual cyclical movement, but always the difference between any year and the same season of the preceding year. These differences can best be seen symbolically.

Older methods

$$\frac{C_{ij+1}}{T_{ij+1}} = \text{Relative Cycle.}$$

Direct method

$$\frac{C_{ij+1} - C_{ij}}{O_{ij}} = \text{Relative cyclical difference.}$$

Manifestly these two differ, but let us see just how. The numerator in the second case is algebraically less by the amount C_{ij}, whereas the denominator in the second case is greater by $S_{ij} + C_{ij} + k_j$. But, from this it does not follow that the second case will give relatives smaller than the first case because both S and C are taken algebraically. Hence all sorts of relationships between the old and the direct seem to be possible. These possible relationships are not quite so heterogeneous when we consider, whatever direction C_{ij} takes, O_{ij} will swing in the same direction. And it is this fact which insures regularity in the indices in the second case. The existence of this regularity will

become more apparent in the next chapter when the method is applied to actual figures. It will appear from study of the data that the fluctuations of the new method are greater than those of the old, and that the new method is more sensitive to irregularities than the old.

C. Conclusions

Rather than attempt to recapitulate the criticisms of pre-existing concepts and of previous methods, we shall bring this chapter to a close with a restatement of the steps in the direct method of cyclical representation which is presented.

1. Divide each item of original data by the item for the same season of the preceding year. (The relatives will begin one year later than the original data, but can be computed for any recent item as soon as such is available.)

2. Compute the arithmetic average of these relatives. (If the result is greater than or less than 100 the trend residue is positive or negative respectively. If the result equals 100 there is no trend.)

3. The variation from 100 of the average above represents trend residue. Divide the arithmetic average of the original data by each item of original data and multiply the results by the trend residue. This latter is the correction to remove the trend residue which is applied to each relative obtained in the first step.

4. The remainders after the subtraction of this correction are the relatives of cyclical differences which we set out to obtain.

III

Application of the Direct Method to a Controlled Set of Data

Construction of the Data.—We can now turn to the more concrete problem of testing the direct method with sets of data. This chapter will be devoted to a study of a set of data which is completely controlled in the sense that the components have been superimposed upon each other, so that the part of each item which is to be attributed to trend, seasonal variation and cyclical fluctuation, respectively, is known in advance and independently of any method of obtaining them from the completed data itself. The purpose may be illustrated by an analogy. Let us suppose a chemist desires to test the accuracy of a method of chemical analysis. He prepares a compound of ingredients A, B and C in certain known quantities or proportions. The method in question is applied and if analysis results check with the quantities or proportions used in the synthetic process the method is considered satisfactory.

It will not be necessary to introduce any great amount of detail concerning the method of constructing such a set of data, because the device is well known. Assuming that the data represent something which has been in existence for some length of time we give it a certain initial volume. From this starting point a trend is introduced in the form of a straight line. On this line is superimposed an harmonic function of fixed amplitude and period. This component purports to be the cycle. Just what amplitude it shall have is at the choice of the manipulator, likewise its period, although the latter may be made to agree with *a priori* knowledge of the period of the cycle in actual data. The third component is the seasonal, an harmonic function with period one year and amplitude at the discretion of the manipulator.

Let us note the characteristics of this set of data. There are no irregular movements, trend is a straight line, seasonal varia-

tion repeats itself with equal *numerical* intensity from year to year, the cycle is regular both as to amplitude and period. It is precisely because no such set of data as this exists in the economic world, that we are unable in practice to arrive at any such clear-cut distinction between movements as we have here, although by the use of artificial devices such as the present one we can construct an assumed case. This is the situation on which we based the statement of Chapter I to the effect that the introduction of mathematical exactness into the representation of economic time data involved the assumption of more regularity of causation in the fluctuations being measured than experience warranted. But neither our faith in, nor our knowledge of the direct method can on that account be adversely affected by its *successful* application to the proposed controlled data. In other words, if the present method does not meet the test of correctly analyzing a controlled set of data it must be rejected immediately. If it successfully undergoes the test of controlled data this increases our estimate of its dependability, but the further tests of actual conditions will then be applied.

The equation of the data chosen is:

$$Y = 500 + 4X + 100 \sin \frac{\pi X}{20} - 20 \sin \frac{\pi X}{6}; \text{ for } X = 0 \text{ to } X = 119$$

The volume of the data at the beginning of the period considered is 500. The time period covered is 10 years of monthly data. The three components follow in order. 4X gives the trend amount for the various values of X. $100 \sin \frac{\pi X}{20}$ gives a periodic function whose period is 40 months. This length of time was chosen as the period of the cycle, because it is nearest to being in line with the results of other theories. We will then have three complete cycles in the 10-year period. The term $-20 \sin \frac{\pi X}{6}$ is the seasonal component with period 12 months. The constants chosen for determining the amplitudes of these transcendental functions and for determining the slope of the

TABLE I

CONTROLLED DATA. COMPUTATION OF RELATIVE CYCLICAL DIFFERENCES

	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH
J	500	643.1 128.6 121.7	537.2 83.5 75.2	585.2 108.9 101.3	787.1 134.5 128.8	740.0 94.0 88.0	692.9 93.6 87.2	894.8 129.1 124.1	942.8 105.4 100.7	836.9 88.8 83.5
F	509.6	631.1 123.8 116.7	519.3 82.3 73.7	592.6 114.1 106.6	784.8 132.4 126.7	718.4 91.5 85.3	692.9 96.5 90.1	900.7 130.0 125.1	923.4 102.5 97.7	827.2 89.6 84.2
M	521.6	619.6 118.8 111.6	505.8 81.6 72.8	603.8 119.3 111.9	782.7 129.6 123.9	699.8 89.4 83.0	697.8 99.7 93.3	906.6 129.9 125.0	905.6 99.9 95.0	822.7 90.8 85.4
A	537.4	610.7 113.6 106.4	498.9 81.7 72.8	620.4 124.4 117.2	782.8 126.2 120.5	686.6 87.7 81.2	709.3 103.3 97.0	917.1 129.3 124.4	891.6 97.2 92.2	825.2 92.6 87.2
M	557.5	605.5 108.6 101.2	499.6 82.5 73.6	642.7 128.6 121.7	785.8 122.3 116.6	679.9 86.5 80.0	727.9 107.1 101.0	929.8 127.7 122.9	882.7 94.9 89.9	835.6 94.7 89.4
J	580.7	603.4 103.9 96.5	507.2 84.1 75.3	669.6 132.0 125.4	791.1 118.1 112.5	679.3 85.9 79.3	752.6 110.8 104.9	944.8 125.5 120.8	878.4 93.0 87.9	852.9 97.1 91.9

TABLE I (*Continued*)

	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH
J	604.9	602.9 99.7 92.3	520.0 86.2 77.6	698.9 134.4 128.0	796.9 114.0 108.4	683.1 85.7 79.2	781.1 114.3 108.6	960.0 122.9 118.3	877.1 91.4 86.3	875.1 99.8 94.7
A	627.1	601.6 95.9 88.5	535.2 89.0 80.7	727.4 135.9 129.8	800.7 110.1 104.5	688.9 86.0 79.5	810.4 117.6 112.1	972.8 120.0 115.4	876.6 90.1 85.0	899.3 102.6 97.6
S	644.4	597.3 92.7 85.2	550.2 92.1 84.0	752.1 136.7 130.8	800.1 106.4 100.8	694.2 86.8 80.4	837.3 120.6 115.3	980.4 117.1 112.6	874.5 89.2 84.1	922.5 105.5 100.7
O	654.8	588.4 89.9 82.3	562.9 95.7 87.8	770.7 136.9 131.1	793.4 102.9 97.3	697.2 87.9 81.5	859.6 123.3 118.1	981.1 114.1 109.6	869.3 88.6 83.5	942.6 108.4 103.7
N	657.3	574.4 87.4 79.6	572.4 99.7 91.9	782.2 136.7 131.0	780.3 99.7 94.0	697.3 89.4 83.0	876.2 125.7 120.6	974.2 111.2 106.6	860.4 88.3 83.1	958.4 111.4 106.8
D	652.8	556.6 85.3 77.3	579.3 104.1 96.4	787.1 135.9 130.2	761.6 96.8 91.0	695.2 91.3 84.9	887.4 127.6 122.6	960.7 108.3 103.7	848.9 88.4 83.2	970.4 114.3 109.7

trend line are purely arbitrary. The sole criterion of their choice is an attempt to make the example reasonable.

It may be well to show the computation of a few of the items in the hypothetical data to which the method is applied in Table I. If we substitute the indicated values of X in the equation above we have,

$X = 0 \quad Y = 500$

$$X = 1 \quad Y = 500 + 4 + 100 \sin\frac{\pi}{20} - 20 \sin \frac{\pi}{6} = 509.6$$

$$X = 2 \quad Y = 500 + 8 + 100 \sin \frac{\pi}{10} - 20 \sin \frac{\pi}{3} = 521.3$$

: : : : : : :
: : : : : : :
: : : : : : :
: : : : : : :
: : : : : : :

$$X = 119 \; Y = 500 + 476 + 100 \sin - \frac{\pi}{20} - 20 \sin - \frac{\pi}{6} = 970.4$$

These operations furnish the original data for the test, which appears in Table I as the top figure in each block. The entire operation of computing the cycles from these data is represented in Table I so for each step of the computation one figure is to be read from each block of the Table.

Application of Direct Analysis to the Data.—The first step in the segregation of cycles is to divide the items of the second year by the corresponding items of the first year, etc., *i.e.*,

$$\frac{643.1}{500} = 128.6; \frac{631.1}{509.6} = 123.8; \frac{619.6}{521.3} = 118.8; \ . \ . \ .$$

and finally $\frac{970.4}{848.9} = 114.3$. The next step is to take the average of these relatives. This is $\frac{11439.3}{108} = 105.9$. So that 5.9 is the average upward tendency introduced by the trend residue k_j. We now proceed to distribute this amount. Multiplying the arithmetic average of the original data, excluding the first year,

754.7 by 5.9 we have 4452.73. And by dividing this by each item of original data except the first year, we get the amount to be subtracted from each relative in order to get the final *relative cyclical differences*. The results of this operation appear as the lowest figure in each block of Table I.

Interpretation of the Results. The Nature of the Trend Removed.—This completes the application of the method to the assumed facts, so we turn to the study and interpretation of the results. The first question which presents itself is whether the relatives obtained actually express cyclical differences. As an answer to this question Table II is presented in which the upper figures are the variations of the relatives of Table I from 100 multiplied by the item of original data for the same season of the preceding year according to the equation

$$C_{ij+1} - C_{ij} = H \cdot O_{ij}$$

for example, the first computation is $25.7 \cdot 500 = 128.5$. The lower figures are the actual figures for the difference $C_{ij+1} - C_{ij}$ where by actual is meant the cyclical values obtained from solution of the first harmonic term of the original equation. The two sets of values in Table II do not agree, but their differences are small and, as the test requires, follow each other closely.

Two primary facts must be noted in connection with the discrepancies between these two sets of differences. The lower set was obtained from the use of straight-line trend. The direct method which was used to analyze the data does not assume straight-line trend. Thus the discrepancies which appear between the lower and upper sets of figures express the amounts by which the trend residues of the direct method vary from straight-line trend increments. Hence the smallness of these differences very definitely establishes two points—that we have succeeded in removing trend without being forced to make any inelastic assumption concerning its nature, and that, inasmuch as the trend of the present controlled experiment is something approaching a straight line, the method used has succeeded in removing just that.

The Adjustment to Changes in the Nature of Both the Seasonal and Cyclical Components.—The second point to

TABLE II

VARIATIONS BETWEEN ACTUAL AND COMPUTED CYCLICAL DIFFERENCES

	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH
J		108.5 95.1	159.5 153.9	6.9 0	168.5 153.9	94.4 95.1	94.7 95.1	167.0 153.9	6.3 0	155.6 153.9
F		85.1 73.5	166.0 159.8	34.3 25.3	158.2 144.2	115.4 114.4	71.1 73.5	173.9 159.8	20.7 25.3	145.9 144.2
M		60.5 50.0	168.5 161.8	60.2 50.0	144.3 130.9	133.1 130.9	46.9 50.0	174.4 161.8	45.3 50.0	132.2 130.9
A		34.4 25.3	166.1 159.8	85.8 73.5	127.2 114.4	147.2 144.2	20.6 25.3	173.1 159.8	71.5 73.5	114.1 114.4
M		6.7 0	159.9 153.9	108.4 95.1	106.7 95.1	157.2 153.9	6.8 0	166.7 153.9	93.9 95.1	93.6 95.1
J		20.3 25.3	149.0 144.2	128.8 114.4	83.7 73.5	163.8 159.8	33.3 25.3	156.5 144.2	114.3 114.4	71.2 73.5
J		46.6 50.0	135.0 130.9	145.6 130.9	58.7 50.0	165.8 161.8	58.7 50.0	142.9 130.9	131.5 130.9	46.5 50.0
A		72.1 73.5	116.1 114.4	159.5 144.2	32.7 25.3	164.1 159.8	83.4 73.5	124.8 114.4	145.9 144.2	21.0 25.3
S		95.4 95.1	95.6 95.1	169.5 153.9	6.0 0	156.8 153.9	106.2 95.1	105.5 95.1	155.9 153.9	6.1 0
O		115.9 114.4	71.8 73.5	175.1 159.8	20.8 25.3	146.8 144.2	126.2 114.4	82.5 73.5	161.9 159.8	32.2 25.3
N		134.1 130.9	46.5 50.0	177.4 161.8	46.9 50.0	132.6 130.9	143.6 130.9	57.8 50.0	164.6 161.8	58.5 50.0
D		148.2 144.2	20.0 25.3	174.9 159.8	70.8 73.5	115.0 114.4	157.1 144.2	32.8 25.3	161.4 159.8	82.3 73.5

which attention is called is the fact that a rather peculiar situation was introduced into the problem by the method of constructing the data. The values attributed by assumption to cyclical fluctuation do not increase with the natural increase in the volume of the data. This fact gives us in reality a set of data of increasing volume in which the importance of the cyclical swings is decreasing. So that the problem solved not only is a test of accuracy on an ordinary case but furnishes an answer to the further question as to whether the method will care for changes in the nature of the cyclical movements. A study of the two sets of figures in Table II shows that the variations between the two are of the same type at the end of the period as they were at the beginning, which demonstrates the ability of the method to adjust itself to changes in the *nature* of the cyclical fluctuations. Further verification of this feature of the method can be obtained by studying the relatives of cyclical differences of Table I. The swings are less violent from cycle to cycle. We have for the highs, October of the 4th year 131.1; February of the 8th year, 125.1; for the lows, March and April of the 3rd year, 72.8; July of the 6th year, 79.2; November of the 9th year, 83.1. This is very plainly shown in the graphic comparison of the results.

We can likewise see that the seasonal variations of the original data remain constant in absolute amount from year to year. In face of the increasing volume of the total data this is essentially a change in the nature of the seasonal variation. And once more the method has easily adapted itself to the change as is shown by the lack of any progression in the divergence between the figures of the upper and lower sets of Table II.

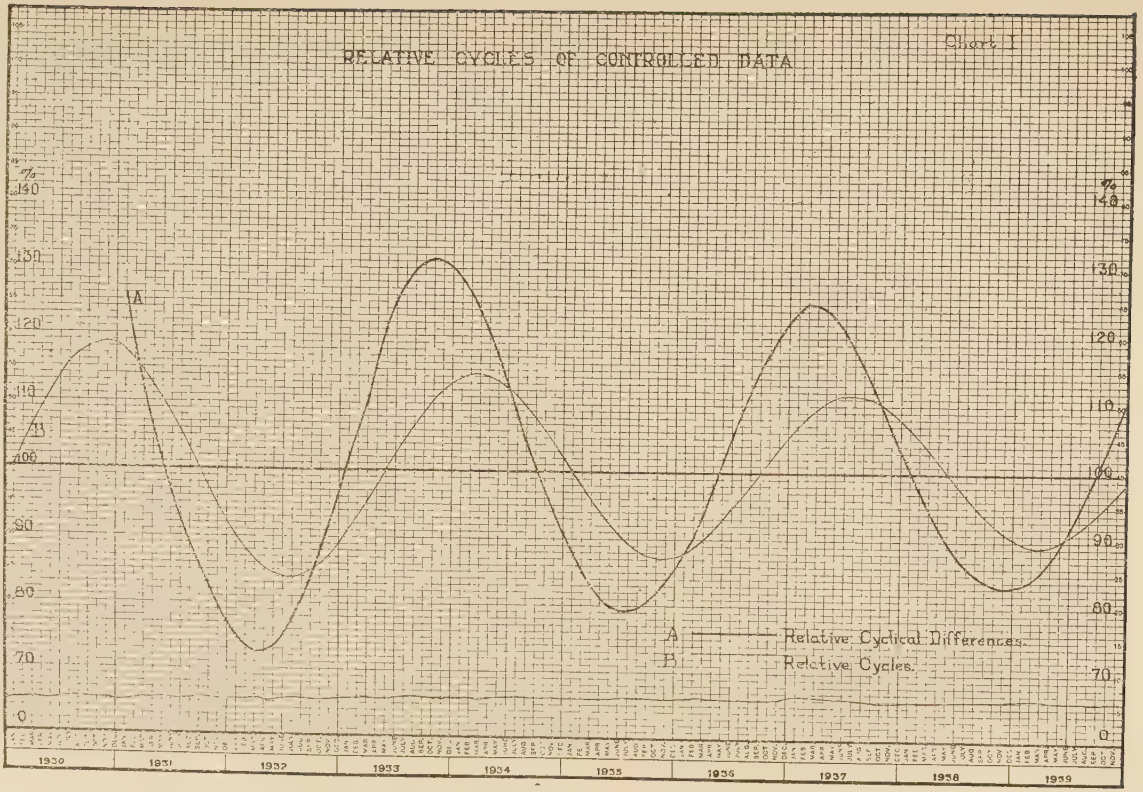
Comparison of the Cyclical Differences of the Direct Method with the Cycles Assumed in the Problem.—Turning to the relatives of cyclical differences of Table I, we enter upon the interpretation of the primary result. As predicted in the discussion of the preceding chapter, these relatives are entirely regular in their oscillations, consequently they possess the characteristic of relative cycles as we are accustomed to find them in the older methods. There are, then, two questions for study:

1. What is the relationship between the *amplitudes* of the cyclical relatives of the new method and those assumed in the original data?

2. What is the relationship between the *periods* of the cyclical relatives of the new method and those assumed in the original data?

The answers to these questions can be more readily ascertained by supplementing the figures with a graphic presentation of the two sets of relatives. In Chart I the assumed cyclical values of the original data are reduced to relatives of the trend and plotted from 100 as a base, giving Curve B of the chart. The relative cyclical differences obtained by the direct method are plotted directly from Table I, giving Curve A of the chart. The latter, of course, starts one year later than the period covered by the data itself. Attention may now be especially called to the point made earlier that these data contain in reality a cyclical component of relatively decreasing magnitude. Both the A and B curves show this very clearly. The amplitudes of successive cycles in both the positive and negative phases grow progressively smaller.

The amplitudes of the cyclical swings of Curve A are greater than those of Curve B, because the algebraic difference of two cyclical magnitudes is related to the total data in A, whereas a single cyclical magnitude is related to the corresponding trend value in B. The result is greater not merely by accident, but because the two cyclical magnitudes whose difference is taken are in general of opposite sign, thus giving a greater numerator than in the case of the values for Curve B. It is fairly apparent that the greater the percentage of the total volume of any set of data which arises from cyclical fluctuation, the greater the relatives of cyclical differences will be. As a limiting case we might conceive of a set of data in which no movement was operative except the cycle. In such a set the relatives would fluctuate violently, and indeed if the volume of the data should be near zero at any time, the relative of the year following would approach either a positive or a negative infinity. The reader may feel that this is a rather fantastic concept to introduce,


RELATIVE CYCLES OF CONTROLLED DATA
Chart I
%
140
130
120
110
100
90
80
70
0
A
B
A Relative Cyclical Differences.
B Relative Cycles.
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939

since the probability of its occurrence in the economic world is approximately zero. On the other hand, if it is felt that this constitutes a valid objection to the method, we may state that the results either of this method or the older ones will not be affected from the standpoint of interpretation or comparison, if a fixed amount be added to each item of original data. In the direct method this will result in stabilizing the relative cyclical differences. In the older methods the trend values will be increased and all relative cycle values will be decreased. Hence this device might well be used in connection with either the direct method or the older methods in those cases in which the cyclical fluctuations were too violent for ease of interpretation. Of course, it sacrifices amplitudes of cycles but it preserves all the relationships between relative cyclical fluctuations. There is no great virtue in obtaining the absolute amplitude of a particular cycle in any event, for the only valuable interpretation arises from comparison of one cyclical swing with another. So we conclude with respect to question (1) that the amplitudes of cyclical differences are not the same as the amplitudes of cycles, but that the relationship *between* amplitudes of different cycles by the one method is the same as by the other.

We turn to Chart I to study the relationship between the periods of the cycles by the two methods. Curve B, of course, must show exactly what it was defined to be in the beginning—three completed cycles each of 40 months' duration. Curve A does not show three complete cycles because it starts one year later than B. Studying it in terms of the points at which it cuts the 100 value, we have the first intersection in May of the second year, the next in December of the third year, the next in September of the fifth year, the next in April of the seventh year, the next in January of the ninth year and the last in August of the tenth year. These crossings are 20 months apart, so that the period of the relatives of cyclical differences is exactly the same as that obtained by using the relative cycles. Close observation of the chart will disclose a fact which is confirmed by looking at the relatives of Table I; that when the cycle is changing from positive to negative the curve crosses 100 slightly after the beginning of the month, and when it is changing from negative

to positive the curve crosses 100 slightly before the end of the preceding month. Hence the positive phase of the cycle is slightly longer than 20 months and the negative phase is slightly shorter than 20 months. This is not a phenomenon requiring interpretation. The reason for it was explained in Chapter II.[1] In removing the trend residue, k_j, we used an arithmetic average of the relatives $(1 + D)$ to obtain the amount of trend influence to be removed and we finally subtracted the adjusted values. Theoretically a geometric average should have been used and a division performed instead of a subtraction. The slight discrepancies occurring here will show with what justice the statement was made at that time that the sacrifice of accuracy was warranted in order to avoid an enormous increase in computation.

Relative Cyclical Differences Precede Relative Cycles. —By this time the main attention of the reader has doubtless centered on the fact that Curve A moves ahead of Curve B. The question is, can this be interpreted as a persistent tendency, or is it an accidental concomitant of the particular set of data with which we are working? It is not accidental. In this problem every movement of the A curve is exactly four months prior to the corresponding movement of the B curve. The reason for this is bound up with the period of the cycle, and the general conclusion concerning it is that the longer the period of the cycle the greater the time by which the cyclical differences precede the actual cycles and vice versa. The logic of this is fairly evident. Suppose we have a two-year cycle. Then the high and the low items would be exactly one year apart, and the maximum relative cyclical difference would coincide with the maximum relative cycle. Further the curve would change from a positive to a negative phase and vice versa at intervals exactly one year apart, so that the relative cyclical differences would cross 100 at the same time as the relative cycles. Hence, for a two-year cycle the two methods would coincide in the time of their movements.

If the length of the cycle is greater than two years the maximum

[1] See page 20.

point and the minimum point of a cycle will be more than one year apart, consequently the greatest difference will not occur when the cyclical value is greatest. It will come when the cyclical value is increasing fastest on the positive phase, and when it is decreasing fastest on the negative phase. The rate of change is retarded near the top of a cyclical swing and likewise near the bottom, and for this reason the cyclical differences reach their maxima and minima in advance of the maximum and minimum values of the cycles themselves. The longer the period of the cycle, the longer the retardation or inertia period near the end of any phase of the cycle, and as a consequence the greater the time by which the maximum cyclical difference precedes the maximum or minimum cyclical value.

The four months period, then, of the data at hand is peculiar to it, but the tendency is inherent in the method. As long as we are dealing with cycles of period greater than two years the cyclical differences will precede the cycles in their movements. If a cycle of less than two years' duration is encountered, the reverse will occur. It is highly probable that no more specific interpretation can be given to this characteristic in advance of the application of the method to particular sets of data. Its function in connection with a particular study will appear in the next chapter.

A Further Experiment.—The manner of construction of the controlled data allows us to make another test of the operation of the direct method. Let us omit from the data the cyclical harmonic, $100 \sin \frac{\pi X}{20}$. We then have items which are composed of the initial volume of the data, the trend growth and the seasonal harmonic. These items appear as the top figures of each block of Table III. Applying the first step of the method to this set we obtain the relatives in the middle positions of the several blocks. These relatives have an average value of 107, hence the trend residue $k_j = +7$. The average of the items in the top positions is 761.8. We remove the trend residue by subtracting from each of the middle relatives the distributed value of k_j. By distributed, of course, we mean the result of

taking the ratio of the average of the items to each of the items and multiplying the result by k_j. The remainders after this operation appear in the lowest position of each block of the Table.

If the method fulfils its purpose when applied to this set of data the final result should be indices which exhibit no variations, since the data contain no cyclical component. It was stated earlier that all changes arising from the presence of trend and seasonal influences were included in the base, 100, in the direct method, hence relatives above and below 100 expressed cyclical differences. It follows that the final results of solving this problem should be relatives of 100 in every case. The lowest figures in each block of the Table demonstrate the extent to which the example verifies the theory. The relatives are in every case less than 100, the lowest one being 99.6. The negligible character of this difference leads to the conclusion for this set of data at least, that the direct method not only removes both trend and seasonal influences, but that it removes *all* of such influences.

The fact that all of these relatives are less than 100 can be accounted for readily. When we compute an arithmetic average, emphasis is placed on the large items of the set. The difference between the large and small items is due to the trend growth, but there is no effect of this growth in the relatives which are averaged to obtain the trend residue. Hence the arithmetic average which is the numerator of the ratio on which the distribution of the trend residue is based is too large,[2] and too much is removed leading to the relatives of the Table which are uniformly less than 100. This situation focuses attention once more on the point discussed in Chapter II,[3] that the use of the arithmetic average introduces a bias in the results, but that the bias is of a negligible magnitude.

The application of the direct method to this set of data leads to one further conclusion. No spurious cycles are indicated where clearly none existed. This result, while negative in

[2] In the sense that it accents the large items of the set.
[3] Page 20.

TABLE III

CONTROLLED DATA WITH CYCLICAL HARMONIC OMITTED. COMPUTATION OF RELATIVE CYCLICAL DIFFERENCES

	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH
J	500	548 109.6 99.9	596 108.8 99.9	644 108.1 99.8	692 107.5 99.8	740 106.9 99.7	788 106.5 99.7	836 106.1 99.7	884 105.7 99.7	932 105.4 99.7
F	494	542 109.7 99.9	590 108.9 99.9	638 108.1 99.7	686 107.5 99.7	734 107.0 99.7	782 106.5 99.7	830 106.1 99.7	878 105.8 99.7	926 105.5 99.7
M	490.7	538.7 109.8 99.9	586.7 108.9 99.8	634.7 108.2 99.8	682.7 107.5 99.7	730.7 107.0 99.7	778.7 106.6 99.7	826.7 106.2 99.7	874.7 105.8 99.7	922.7 105.5 99.7
A	492	540 109.8 99.9	588 108.9 99.8	636 108.2 99.8	684 107.5 99.7	732 107.0 99.7	780 106.5 99.7	828 106.1 99.7	876 105.8 99.7	924 105.5 99.7
M	498.7	546.7 109.6 99.8	594.7 108.8 99.8	642.7 108.1 99.8	690.7 107.5 99.8	738.7 106.9 99.7	786.7 106.4 99.6	834.7 106.1 99.7	882.7 105.8 99.8	930.7 105.4 99.7
J	510	558 109.4 99.8	606 108.6 99.8	654 107.9 99.7	702 107.3 99.7	750 106.8 99.7	798 106.4 99.7	846 106.0 99.7	894 105.7 99.7	942 105.4 99.7

TABLE III (*Continued*)

	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH
J	524	572 109.2 99.9	620 108.4 99.8	668 107.7 99.7	716 107.2 99.8	764 106.7 99.7	812 106.3 99.7	860 105.9 99.7	908 105.6 99.7	956 105.3 99.7
A	538	586 108.9 99.8	634 108.2 99.8	682 107.6 99.8	730 107.0 99.7	778 106.6 99.7	826 106.2 99.7	874 105.8 99.7	922 105.5 99.7	970 105.2 99.7
S	549.3	597.3 108.7 99.8	645.3 108.0 99.7	693.3 107.4 99.7	741.3 106.9 99.7	789.3 106.5 99.7	837.3 106.1 99.7	885.3 105.7 99.7	933.3 105.4 99.7	981.3 105.1 99.7
O	556	604 108.6 99.8	652 107.9 99.7	700 107.4 99.8	748 106.9 99.8	796 106.4 99.7	844 106.0 99.7	892 105.7 99.7	940 105.4 99.7	988 105.1 99.7
N	557.3	605.3 108.6 99.8	653.3 107.9 99.7	701.3 107.3 99.7	749.3 106.8 99.7	797.3 106.4 99.7	845.3 106.0 99.7	893.3 105.7 99.7	941.3 105.4 99.7	989.3 105.1 99.7
D	554	602 108.7 99.8	650 108.0 99.8	698 107.4 99.8	746 106.9 99.8	794 106.4 99.7	842 106.0 99.7	890 105.7 99.7	938 105.4 99.7	986 105.1 99.7

character, serves to clear away any doubt on the point of whether the relative cyclical differences are free from residuals from the eliminated components.

Conclusions.—The application of the method developed in Chapter II to a controlled set of data has served three major purposes.

1. The solution of a problem affords a much clearer idea of the simplicity of the direct method, and at the same time gives a better idea of the details of its operation.

2. Solution of the problem provides an opportunity for comparison between the relatives of cyclical differences by the direct method and the relative cycles of previous methods. This comparison allows us to state that the relationship between amplitudes of various cycles is the same for the new and old methods, although the values of amplitudes are different; further, that periods of cycles are maintained unaltered; and that the relatives of cyclical differences precede the relative cycles in their movements for any cycle whose period is greater than two years, the time of this precedence being in general variable.

3. Finally it is established that the method proposed for the elimination of trend and seasonal variation without computing them has actually accomplished its purpose. Whether the same conclusions will result from the application of the direct method to uncontrolled data will appear in Chapter IV.

IV

Application of the Direct Method to Actual Data

Explanation of the Data Employed.—Three sets of data will be used for illustrative purposes in this chapter. The choice of these is largely optional from the standpoint of theory, but in practice they have been selected on the basis of their peculiarities rather than their regularity. The purpose of this is to afford a more complete test of the method than would otherwise be accomplished. Even if we could find a set of data which possessed regularity of movements approaching anywhere near the controlled data of Chapter III, there would be small virtue in its use, (1) because we have already seen the results for regular data, and (2) because we are well aware that such a set would not usually be representative of actual business conditions. The preferable thing has seemed to be the opposite extreme, to select sets which are not only representative of business generally but which present a large number and variety of its peculiar movements. No attempt will be made to list these eccentricities at this point, but merely a statement of the *a priori* basis on which the data were chosen will be presented.

1. Bradstreet's Index of Commodity Prices has little or no seasonal variation, hence it will serve to show the results of applying the method to non-seasonal data.

2. Number of Business Failures is largely irregular in its cyclical fluctuations, consequently it will exemplify the power of the method to show this randomness. This last statement may seem to be at variance with common notions of the movements of business failures. If so, we may merely note at this point that only an *a priori* judgment has been made concerning the nature of these movements. It will be better to join the issue after the data have been analyzed.

Number of Business Failures introduces a further feature worth considering. During the war period when data for both production and prices tended to have decided upward swings, the number of failures was at a low ebb, and in general in all economic periods whatever regularity of cyclical movement may be attributed to number of failures will invariably be in inverse direction to Commodity Prices. For this reason Business Failures afford a further opportunity to test the method.

3. Carload Shipments of Automobiles from the Factories have more than trebled themselves in the last twelve years, and there is a marked difference in number of carloads shipped at different seasons of the year, hence this series provides a test of the ability of the method to remove a very steep trend and a high seasonal variation simultaneously.

Another point of adverse selection concerns the period for which the data were taken. Bradstreet's Prices and Business Failures cover in monthly items the years 1911 to 1925 inclusive, Automobile Shipments start three years later in 1914. The first two could have been carried back farther, if there had seemed to be any advantage in so doing, but since the purpose here is merely an illustrative test, no advantage is apparent. In the case of Automobile Shipments, authentic figures were not available for the years prior to 1914. The main purposes of taking this particular period were, first, to include pre-war, war and post-war conditions, thereby giving a test more severe than commonly occurs, and secondly, to demonstrate the important point hitherto passed over, that cyclical movements can be computed as soon as the data are available. Certain details connected with this last point will naturally occur to the reader, but their discussion will be greatly facilitated by reference to actual figures, hence they are discussed along with the analysis of the data.

The data have been taken from two sources. Bradstreet's Index Number and Number of Business Failures are taken from *Bradstreet's Journal*, and Automobile Shipments from the *Statistical Bulletin* of the Standard Daily Trade Service. The latter data are collected by the National Automobile Chamber of Commerce of New York City.

Application of the Direct Method to the Data

Bradstreet's Index Number

Explanation of the Construction of Table IV.—The figures at the top of each block of Table IV are the items of original data. The general tendency is a fairly regular growth from 1911 to 1915, strong increase from 1916 to 1920, a precipitate decline in the latter part of 1920 continuing through 1921 and a very gradual growth since 1922. This may well be considered as an adequate empirical description of the trend for the period. It is certainly not a description of the cycle, as may readily be ascertained by inspection of the years 1922 to 1925. There are consistent differences of as much as 26 per cent between items of consecutive years during this period, yet the description above designates the general tendency of this period as very gradually increasing.

The middle figures in each block of Table IV are the results of dividing the items for the same month of successive years. That is, $\frac{8.95}{8.84} = 101.2$; $\frac{8.96}{8.77} = 102.2$; and so for each pair. These relatives contain a residue of trend so that their sum is 17850.2 which gives an average relative of $\frac{17850.2}{170} = 105$. The trend residue designated earlier as k_j is $105 - 100 = 5$. The next step consists of distributing this residue in proportion to the volume of the original data. The arithmetic average of the data is 13.19, hence the correction would be as follows: For the first item $\frac{13.19}{8.95} \cdot 5 = 7.4$, and $101.2 - 7.4 = 93.8$; for the second item $\frac{13.19}{8.96} \cdot 5 = 7.4$, and $102.2 - 7.4 = 94.8$; and so for each item of the set. The figures at the bottom of each block of the table are the relatives of cyclical differences which we are seeking.

Concerning the Assumptions of the Direct Method.—Before we proceed to the study and comparison of these results let us examine the manipulation in relation to the assumptions of

TABLE IV

BRADSTREET'S INDEX OF PRICES. COMPUTATION OF RELATIVE CYCLICAL DIFFERENCES

	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926
J	8.84	8.95 101.2 93.8	9.49 106.0 99.1	8.89 93.7 86.3	9.14 102.8 95.6	10.92 119.5 113.5	13.73 125.7 120.9	17.94 130.7 127.0	18.53 103.3 99.7	20.36 109.9 106.7	12.66 62.2 57.0	11.37 89.8 84.0	13.70 120.0 115.2	13.27 96.9 91.9	13.93 105.0 100.3	14.01 100.6 95.9
F	8.77	8.96 102.2 94.8	9.46 105.6 98.6	8.86 93.7 86.3	9.66 109.0 102.2	11.14 115.3 109.4	13.94 125.1 120.4	18.05 129.5 125.8	17.63 97.7 94.0	20.87 118.4 115.2	12.37 59.3 54.0	11.42 92.3 86.5	13.72 120.0 115.2	13.20 96.2 91.2	13.89 105.2 100.5	13.72 98.8 94.0
M	8.69	8.90 102.4 95.0	9.41 105.7 98.7	8.83 93.8 86.4	9.62 108.9 102.1	11.38 118.3 112.5	14.14 124.3 119.8	18.05 127.7 124.0	17.22 95.4 91.6	20.80 120.8 117.6	11.87 57.1 51.5	11.60 97.7 92.0	13.93 120.0 115.3	12.90 92.6 87.5	13.84 107.3 102.5	
A	8.52	9.10 106.8 99.6	9.30 102.1 95.0	8.76 94.2 86.7	9.78 111.6 104.9	11.76 120.2 114.6	14.58 124.0 119.5	18.44 126.5 122.9	17.28 93.7 89.9	20.71 119.8 116.6	11.37 54.9 49.1	11.53 101.4 95.7	13.93 120.8 116.1	12.66 90.9 85.7	13.69 108.1 103.3	
M	8.46	9.27 109.6 102.5	9.14 98.6 91.4	8.62 94.3 96.6	9.80 113.7 107.0	11.75 119.9 114.3	15.12 128.7 124.3	18.89 124.9 121.4	17.24 91.3 87.5	20.73 120.0 116.8	10.82 52.2 46.1	11.70 108.1 102.5	13.67 116.8 112.0	12.56 91.9 86.6	13.32 106.1 101.1	
J	8.53	9.10 106.7 99.5	9.07 99.7 92.4	8.62 95.0 87.3	9.74 113.0 106.2	11.69 120.0 114.4	15.47 132.3 128.0	18.98 122.7 119.2	18.09 95.3 91.7	19.88 109.9 106.6	10.62 53.4 47.2	11.90 112.1 106.6	13.38 112.4 107.5	12.29 91.9 86.5	13.62 110.8 106.0	

TABLE IV (*Continued*)

	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926
J	8.59	9.11	8.95	8.66	9.87	11.53	16.07	19.16	18.90	19.35	10.73	12.11	13.09	12.23	13.85	
		106.1	98.2	96.8	114.0	116.8	139.4	119.2	98.6	102.4	55.5	112.9	108.1	93.4	113.2	
		98.9	90.8	89.2	107.3	111.1	135.3	115.8	95.1	99.0	49.4	107.5	103.1	88.0	108.4	
A	8.66	9.16	9.01	8.71	9.82	11.44	16.40	19.09	20.00	18.83	11.06	12.07	12.82	12.62	14.24	
		105.8	98.4	96.7	112.7	116.5	143.4	116.4	104.8	94.1	58.7	109.1	106.2	98.4	112.8	
		98.6	91.1	89.1	106.0	110.7	139.4	112.9	101.5	90.6	52.7	103.6	101.1	93.2	108.2	
S	8.82	9.22	9.10	9.76	9.80	11.78	16.64	19.03	19.47	17.97	11.09	12.08	12.91	12.81	14.09	
		104.5	98.7	107.3	101.4	120.2	141.3	114.4	102.3	92.3	61.7	108.9	106.9	99.2	110.0	
		97.3	91.5	100.5	94.7	114.6	137.3	110.9	98.9	88.6	55.8	103.4	101.8	94.1	105.3	
O	8.81	9.45	9.15	9.24	9.98	12.04	16.81	18.99	19.52	16.91	11.19	12.50	13.10	13.00	14.15	
		107.3	96.8	101.0	108.0	120.6	139.6	113.0	102.8	86.6	66.2	111.7	104.8	99.2	108.8	
		100.3	89.6	93.9	101.4	115.1	135.7	109.5	99.4	82.7	60.3	106.4	99.8	94.1	104.1	
N	8.89	9.48	9.23	8.86	10.38	12.80	17.07	18.89	19.90	15.68	11.35	13.35	13.14	13.35	14.32	
		106.6	97.4	96.0	117.2	123.3	133.4	110.6	105.3	78.8	72.4	117.6	98.4	101.6	107.3	
		99.6	90.2	88.6	110.8	118.1	129.5	107.1	102.0	74.6	66.6	112.7	93.4	96.7	102.7	
D	8.98	9.55	9.23	9.04	10.65	13.66	17.60	19.02	20.18	13.63	11.31	13.78	13.44	13.53	14.41	
		106.3	96.6	97.9	117.8	128.3	128.8	108.1	106.1	67.5	83.0	121.3	97.5	100.7	106.5	
		99.4	89.4	90.6	111.6	123.5	125.1	104.6	102.8	62.7	77.2	117.0	92.6	95.8	101.9	

Chapter II. The seasonal variation was assumed to be not essentially different for the same months of two consecutive years. In the absence of seasonal variation this assumption is approximately inoperative. We have not yet demonstrated the absence of seasonal variation, but as a matter of fact it is wholly indifferent to this method whether there is seasonal variation or not, because correction is made for it automatically. In anticipation of a subsequent section it can be stated that in only one case, June, does the seasonal variation amount to as much as 1 per cent. The seasonal for June is negative, hence any bias which might result from it would appear in the relation between the June relatives and those for May and July. A study of Table IV clearly fails to show any irregularity in a positive direction at this point. This justifies the original description of these data as non-seasonal.

In Chapter II it was stated that the relatives obtained by dividing each item by the item of the preceding year would average 100 for any number of *complete cycles*, provided no trend were present. In applying the method to the data of this chapter, the requirement of completed cycles is entirely neglected. We must consider the question of whether this introduces a significant error. Such error would, of course, be confined to the correction for trend residue. Let us take a hypothetical case. Assume that the period covered by Bradstreet's Index represents a number of complete cycles. Then the average 105 would be subject to no error from this source. Let 20 new items of data be introduced in a positive phase of the cycle with an average relative of 115 (a rather extreme case has been selected purposely). These items are 10 points above the average, so they increase the total of the relatives to 20150.2, and the average to 106. The correction would then be 6 instead of 5. After this extra 1 per cent has been distributed and the subtractions made, a difference of 1.4 per cent is the maximum change that will appear in any of the final relatives. This 1.4 per cent is worth considering only in connection with the values of the relatives themselves. So far as the relationship between the relatives is concerned all of them have been equally affected, with the exception of a fractional part of a per cent embodied in

the differential, which appears in the distribution of the trend residue.

If this error in the value of the relatives appears to be significant, a slight modification of the method will care for it. Compute the relatives according to the method for the greatest number of completed cycles. Then extend the computation to current data with the stipulation that it is subject to an error of less than 2 per cent, and make the correct computation when the cyclical swing is complete. It does not seem that this error will be large enough to warrant any attention in the ordinary case, unless the method is to be applied to data covering a very short period. For example, if only six to ten years were being studied and there were decided trend, the error from this source would increase proportionately. In such case the change suggested above might have to be brought in. This would not involve any modification of the theory nor additional present computation, but it would call for a recomputation of the entire data at the close of a current cyclical swing.

The reader will feel that we have here introduced a very grave fallacy, *i.e.* the possibility of the short-time movements of current data being able to change the trend of data which represents business ten years ago. That possibility is exactly what we wish to admit rather than deny. It was pointed out earlier that the exact volume of any set of data which is to be attributed to trend probably never can be known. There is a certain borderland of fluctuation within which we hope trend actually lies, and we desire to keep that borderland as narrow as possible. As long as the existence of a region of uncertainty is admitted, there is no fallacy in the foregoing. All we are asking is the right of the trend residue to fluctuate mildly within very narrow limits.

Comparison of Relative Cyclical Differences and Relative Cycles From Chart II.—We turn now to direct interpretation of the relatives of cyclical differences. The easiest way to follow the cyclical swings is by reference to a graph, so we turn to Chart II. The heavy line represents the movements of the relatives of cyclical differences by the direct method. The curve

is nearly normal during 1912, swings gently down through 1913, reaches a low point early in 1914, and is again normal by the latter part of 1915. It increases through 1916 to a high point in the summer of 1917 and then declines through 1918 to a low point in the middle of 1919. It recovers to a high point in the spring of 1920 and then drops to its lowest point in May 1921. There is a speedy recovery to a high point at the close of 1922 and finally a rather gentle down-swing and recovery during 1923–24–25, with a falling off at the end of 1925. This same information can be presented with more facility in tabular form. In the second column of Table V the high and low points and the normal of the curve are given. We can now study the length of the cycle from three standpoints. Taking the highs the lengths of the cycles are, respectively, 1st, 63 months, 2d, 31 months, 3d, 33 months, 4th, 31 months. Taking the lows the lengths of the cycles are, 1st, 55 months, 2d, 24 months, 3d, 35 months. Taking the normal the lengths of the cycles are, 1st, 66 months, 2d, 25 months, 3rd, 39 months. From these figures it is difficult if not impossible, to draw any conclusion concerning the length of the cycle. The variations occurring from such different ways of measuring are to be expected. They arise from the fact that there is very little homogeneity in the lengths of time consumed by the same phases of successive cycles. For example, the phase from high to normal varies from 4 months to 10 months, and the phase from normal to high varies from 5 months to 22 months.

The period including the war years has probably had some tendency to destroy the regularity of the cyclical movements so that on this point we have hardly made an ordinary test. As a matter of fact, however, we have no ambition here of determining the length of the cycle. The most definite conclusion which can be drawn is that the modal or most common length of the cycle since the war has been some 30 to 40 months.

For comparative purposes the cyclical fluctuations of Bradstreet's Index Number have been computed by an orthodox method and plotted by the light line of Chart II. In obtaining these cycles straight-line trend was used and the seasonal was computed as a relative of trend, but it was found to be less than 1 per cent in volume in every case except June, which was 1.2

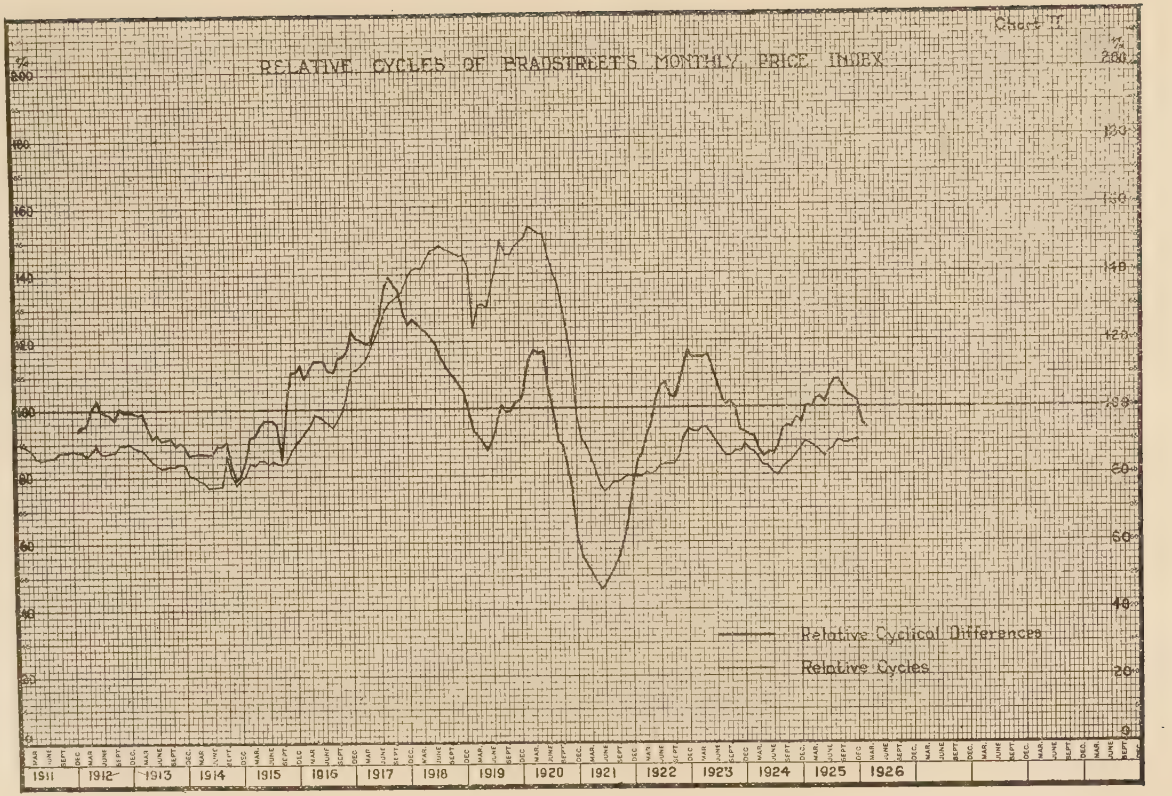


Chart I
RELATIVE CYCLES OF BRADSTREET'S MONTHLY PRICE INDEX
Relative Cyclical Differences
Relative Cycles
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925
1926

per cent. These amounts are all within the region of error of the method of computation, consequently it seemed inadvisable to attempt to remove their influence. The residuals were plotted as relatives above and below 100 so as to make them comparable with the direct method. The striking thing about this second curve is the fact that the cyclical swings above and below 100 as a normal line do not balance. This is a result of the use of straight-line trend. Inspection of the original data of Table IV will show that a straight line is in no sense a fit to the data, but no apology for its use is necessary, because its unrepresentative character is harmless in the absence of seasonal variation. All we desire to draw from this comparison are the relationships between the amplitudes and periods of this latter curve and the first curve of the chart. With proper interpretation these relationships are not biased by the position of the second curve. It will, of course, be apparent to the reader that the first curve has made adjustment with ease for the difficult nature of the trend during the period covered, whereas the second curve is completely unrepresentative of cyclical movements.

Periods.—Taking up first the periods of the two curves we find by reference to Table V that there are some differences between them as shown by the tabulation below.

Length of Cycle

	By Highs		By Lows	
	New Method	Old Method	New Method	Old Method
1st				39 mo.
2nd	63 mo.	67 mo.	55 mo.	54 mo.
3rd	31 mo.	19 mo.	24 mo.	28 mo.
4th	33 mo.	37 mo.	35 mo.	37 mo.
5th	31 mo.			

There are several points, such as the low of the second curve in 1914, the selection of which has been rather arbitrary, and on that account it seems that we are scarcely warranted in drawing too strong a conclusion concerning the agreement of the periods

TABLE V

BRADSTREET'S INDEX OF PRICES. DATES OF THE HIGH, LOW AND NORMAL POINTS OF THE CURVES OF CHART II

	NEW METHOD	OLD METHOD
LOW		MAY, 1911
HIGH	MAY, 1912	DECEMBER, 1912
NORMAL	DECEMBER, 1912	
LOW	OCTOBER, 1914	AUGUST, 1914
NORMAL	OCTOBER, 1915	
HIGH	AUGUST, 1917	JULY, 1918
NORMAL	JUNE, 1918	
LOW	MAY, 1919	FEBRUARY, 1919
NORMAL	OCTOBER, 1919	
HIGH	MARCH, 1920	FEBRUARY, 1920
NORMAL	JULY, 1920	
LOW	MAY, 1921	JUNE, 1921
NORMAL	MAY, 1922	
HIGH	DECEMBER, 1922	MARCH, 1923
NORMAL	OCTOBER, 1923	
LOW	APRIL, 1924	JULY, 1924
NORMAL	JANUARY, 1925	
HIGH	JULY, 1925	

of the two curves. Their general agreement can hardly be disputed. It would be rather disappointing from the standpoint of the new method if they were to agree exactly. We stated earlier that the cyclical differences would precede the cycles by a length of time which varied with the length of the cycle. Since the cycles are quite variable in period during the years studied, the theory requires that the periods of the cyclical differences shall vary, not in exact accord with those of the cycles, but at a

variable rate of divergence, shorter for long cycles, longer for short cycles. The table shows that within the limits of the accuracy of choice of high and low points the data support the theory.

No interpretation could be made in terms of the points at which the two curves are normal, because the second curve crosses 100 at only two points. This third comparison is sacrificed on the altar of straight line-trend.

Amplitudes.—The next consideration will be the amplitudes of the two curves. We stated in Chapter III that *amplitudes* were not preserved by the new method, but that the *relationship between the amplitudes* of successive cycles was preserved. It is not possible to verify that statement from these curves because the second curve is hopelessly distorted by the unrepresentative character of the straight-line trend. But there are certain interpretative features of equal importance to which attention is called. In particular we will study the nature of the two curves in the post-war adjustment period. The second curve shows its highest amplitude in the short up-swing of 1920, whereas the first curve is distinctly higher in 1917. The reason for this is fairly apparent. The curve of cyclical differences shows rate of change in the cycles of the data. The amount of increase of 1920 over the mild down-swing of 1919 does not represent as radical a change as the up-swing of 1916 and 1917 and consequently the first curve does not show as high a peak. That is, the amplitudes of the first curve, as stated earlier, represent the rate of increase or decrease of the data and not the amount resulting from such increase or decrease. This is further shown by the low amplitude of the first curve in 1921. The rate of decline of the data was extremely rapid after the break came. During the years following 1921 the second curve shows only a slight increase in its movements. The first curve shows the fact which the second one omits, that there was a decided positive change in the data in 1922. So we are now prepared to state as a conclusion that the amplitudes of the relatives of cyclical differences represent rates of increase or decrease in the data. When we have a high positive amplitude there has been a high rate of increase in the volume of the data, and when we have a low negative amplitude

there has been a rapid decline in the volume of the data. It must be evident that these amplitudes need not be coincident with either the highest or lowest swings of the relative cycles, if such swings have resulted from gradual growth.

Number of Business Failures

SOLUTION.—These data combine both trend and seasonal variation with the cyclical fluctuations. Hence, the application of both the direct and the older methods to them is in complete form. It is not considered necessary to present any detail concerning the computation of Table VI, since the process is identical with that already explained in connection with Tables I and IV. The major variation which requires attention in this set is its inverse character, *i.e.*, the fact that it reaches its lowest point during the war years. There was no reason to suspect in advance that this feature would present any difficulty, and it turns out in practice that the adjustment for a violent negative swing was made with ease.

COMPARISON OF RELATIVE CYCLICAL DIFFERENCES AND RELATIVE CYCLES. PERIODS.—Next in order is a study of the periods of the two curves of Chart III. The high and low points are presented in Table VII. Some of these points are selected rather arbitrarily, due to the inherent irregularity of the data and still others, such as the low point of the second curve in 1911 and 1912, seemed incapable of determination. The tabulation below is an attempt to compare the periods of the cycles which could be determined.

LENGTH OF CYCLE

	BY HIGHS		BY LOWS	
	NEW METHOD	OLD METHOD	NEW METHOD	OLD METHOD
1st	71 mo.	84 mo.	77 mo.	?
2nd	23 mo.?	22 mo.	32 mo.	44 mo.
3rd			33 mo.?	27 mo.?

TABLE VI

NUMBER OF BUSINESS FAILURES. COMPUTATION OF RELATIVE CYCLICAL DIFFERENCES

	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926
J	1376	1701	1566	1729	2378	1799	1558	1219	573	510	1998	2722	2203	2231	2340	2073
		123.6	92.1	110.4	137.5	75.7	86.6	78.2	47.0	89.0	391.8	136.2	80.9	101.3	104.9	88.6
		114.7	82.4	101.6	131.1	67.3	76.8	65.7	20.5	59.2	384.2	130.6	74.0	94.5	98.4	81.3
F	1012	1208	1114	1206	1865	1608	1126	918	492	432	1435	2072	1563	1572	1654	1700
		119.4	92.2	108.3	154.6	86.2	70.0	81.5	53.6	87.8	332.2	144.4	75.4	100.6	105.2	102.8
		106.8	78.6	95.7	146.4	76.7	56.5	64.9	22.7	52.6	321.6	137.1	65.7	90.9	96.0	93.9
M	1104	1101	1235	1260	1876	1637	1147	893	485	499	1500	2297	1677	1657	1654	
		99.7	112.2	102.0	148.9	87.3	70.1	77.9	54.3	102.9	300.6	153.1	73.0	98.8	99.8	
		85.9	99.9	89.9	140.8	78.0	56.8	61.2	23.5	72.4	290.5	146.5	63.9	89.6	90.6	
A	985	1079	1148	1221	1674	1267	1096	829	459	472	1410	1954	1637	1726	1667	
		109.5	106.4	106.4	137.1	75.7	86.5	75.6	55.4	102.8	298.7	138.6	83.8	105.4	96.6	
		95.4	93.2	93.9	128.0	63.7	72.6	57.3	22.9	70.6	287.9	130.8	74.5	96.6	87.5	
M	1030	1111	1104	1181	1436	1404	1189	792	431	452	1326	1775	1470	1739	1572	
		107.9	99.4	107.0	121.6	97.8	84.7	66.6	54.4	104.9	293.4	133.9	82.8	118.3	90.4	
		94.2	85.6	94.1	111.0	87.0	71.9	47.4	19.1	71.3	281.9	125.3	72.5	109.6	80.7	
J	972	1018	975	1162	1485	1263	1041	747	421	562	1388	1660	1366	1570	1437	
		104.7	95.8	119.2	127.8	85.1	82.4	71.8	56.4	133.5	247.0	119.6	82.3	114.9	91.5	
		89.8	80.2	106.1	117.6	73.1	67.8	51.5	21.8	106.5	236.0	110.4	71.2	105.2	80.9	

TABLE VI (*Continued*)

	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926
J	927	1073	1065	1219	1443	1164	1049	771	417	593	1491	1575	1405	1548	1451	
		115.7	99.3	114.5	118.4	80.7	90.1	73.5	54.1	142.2	251.4	105.6	89.2	110.2	93.7	
		101.5	85.0	102.0	107.9	67.6	75.6	53.8	17.6	116.6	241.2	95.9	78.4	100.4	83.2	
A	942	1020	1085	1191	1275	1304	1001	747	447	617	1642	1675	1440	1432	1353	
		108.3	106.4	109.8	107.1	102.3	76.8	74.6	59.8	138.0	266.1	102.1	86.0	99.4	94.5	
		93.4	92.4	97.0	95.2	90.6	61.6	54.3	25.8	113.4	256.8	93.0	75.4	88.8	83.3	
S	947	995	1108	1367	1267	1173	872	657	426	659	1618	1460	1279	1277	1234	
		105.1	111.4	123.4	92.7	92.6	74.3	75.3	64.8	154.7	245.5	90.2	87.6	99.8	96.6	
		89.8	97.7	112.3	80.7	79.6	56.9	52.2	29.1	131.6	236.1	78.8	75.7	87.9	84.3	
O	971	1093	1252	1445	1349	1265	1005	688	479	861	1809	1602	1608	1578	1407	
		112.6	114.5	115.4	93.4	93.8	79.4	68.5	69.6	179.7	210.1	88.6	100.4	98.1	89.2	
		98.7	102.4	104.9	82.1	81.8	64.3	46.4	37.9	162.0	201.7	79.1	90.9	88.5	78.4	
N	1092	1106	1233	1586	1399	1265	959	549	432	1070	1970	1761	1644	1460	1462	
		101.3	111.5	128.6	88.2	90.4	75.8	57.2	78.7	247.7	184.1	89.4	93.4	88.8	100.1	
		87.6	99.2	119.0	77.3	78.4	59.9	29.5	43.5	233.5	176.4	80.8	84.2	77.8	89.7	
D	1321	1330	1668	2213	1588	1349	1030	519	488	1736	2427	1862	2415	1911	1628	
		100.7	125.4	132.7	71.8	84.9	76.4	50.4	94.0	355.7	139.8	76.7	129.7	79.1	85.2	
		89.3	116.3	125.8	62.2	73.6	61.6	21.1	62.9	346.9	133.5	68.5	123.4	71.2	75.9	

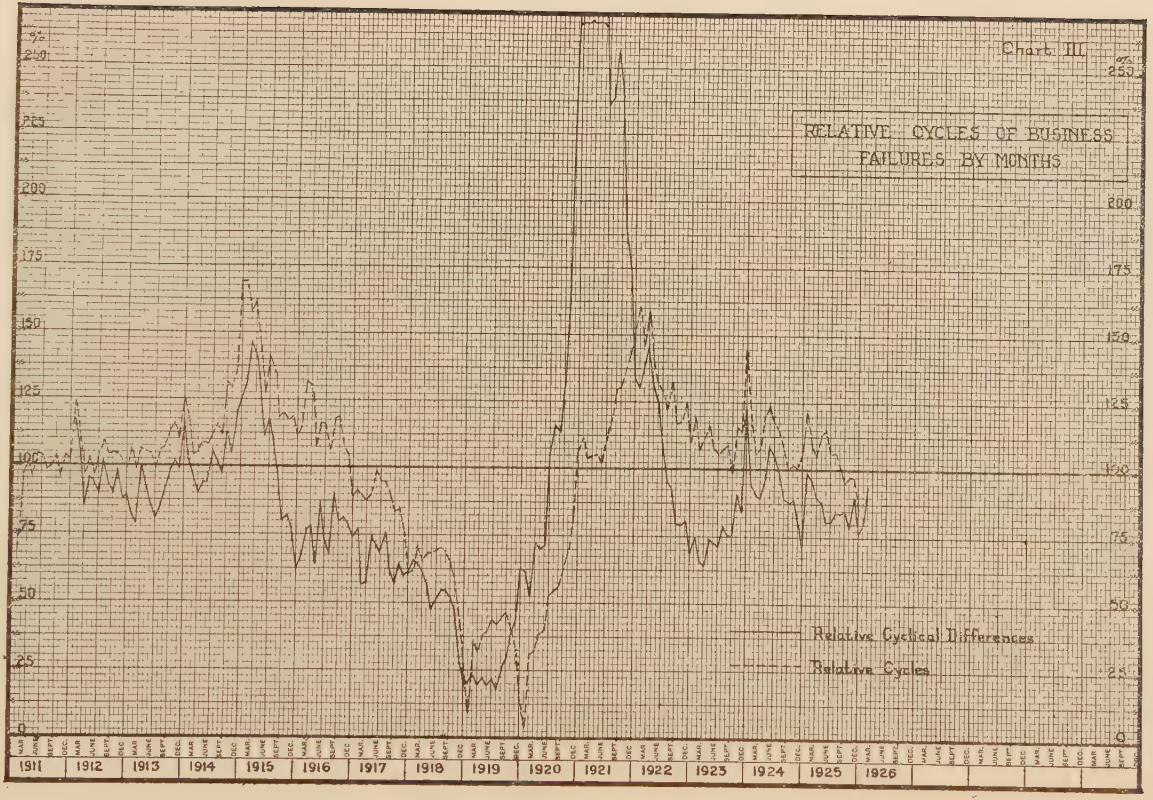


Chart III.
RELATIVE CYCLES OF BUSINESS FAILURES BY MONTHS
%
250
225
200
175
150
125
100
75
50
25
0
Relative Cyclical Differences
Relative Cycles
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925
1926

TABLE VII

BUSINESS FAILURES. DATES OF THE HIGH, LOW AND NORMAL POINTS OF THE CURVES OF CHART III

	NEW METHOD	OLD METHOD
NORMAL	PROBABLY FEBRUARY, 1912	
LOW	FEBRUARY, 1913	IRREGULAR
NORMAL	EARLY IN 1914	
HIGH	FEBRUARY, 1915	JANUARY, 1915
NORMAL	JULY OR AUGUST, 1916	
LOW	JULY, 1919	JANUARY, 1920
NORMAL	JUNE, 1920	
HIGH	JANUARY, 1921	JANUARY, 1922
NORMAL	MAY OR JUNE, 1922	
LOW	MARCH, 1923	SEPTEMBER, 1923
NORMAL	NOVEMBER OR DECEMBER, 1923	
HIGH	POSITIVE SWING NEGLIGIBLE	DECEMBER, 1923
LOW	PROBABLY AT THE END OF 1925	PROBABLY AT THE END OF 1925

There is probably nothing here which warrants drawing a conclusion. The long first cycle cannot be considered as representative because of the war interference. The cycles which have occurred since the war are in general of two to three years' duration. The agreements or variations in period between the old and new methods have very little meaning, because of the arbitrariness which has entered into the choice of the highs and lows. The word choice is used advisedly since the highest point and the lowest point of a particular swing are frequently not representative of it. This is particularly true of the second curve. For example, the relative of January 1920, is less than the

relative of January 1919, but both of these points are a result of the inability of the older method to make adequate compensation for seasonal variations. The seasonal relative for January is +35 per cent. This large value arises from the normal tendency of failures to increase greatly at the beginning of the year. Inspection of the original data for the years 1919 and 1920 in Table VI will show that in neither year was there any such decided increase of failures in January, yet the older method has faithfully deducted 35 per cent with the results shown on the graph. In all probability the low point of this cycle occurred in neither January but somewhere in the fall of 1919 or the spring of 1920. This particular case has been selected as an example of the inaccuracies which result from the older method, when the seasonal variations take on any appreciable degree of irregularity. The same items will serve to demonstrate that the new method is not disturbed by the irregularity of these January seasonal fluctuations. If the reader is inclined to feel that the foregoing might be accidental, let attention be directed to the positions of the second curve for December 1918 and December 1919. The seasonal relative for December is + 21 per cent and this amount has been subtracted, although no change of the sort occurred. The results combine with January to give the two low points of the curve. It is submitted here that the two are not representative of the cyclical movements which occurred at this time.

Amplitudes.—In the light of these facts concerning the failure of the older method adequately to remove seasonal influences, and a point not previously referred to, that the straight line trend is unduly influenced by the small volume of the data during the years 1918, 1919 and 1920, thereby giving cyclical values which are too high in the pre-war and post-war years, it does not appear to be advisable to attempt to compare amplitudes of the two curves. A study can be made of the first curve in terms of its ability to express rates of change in the data. The most rapid decline of the war period occurred in 1918 as compared with 1919, and this is the point at which the curve decreases fastest. The most rapid growth occurred in the second half of 1920 as compared with 1919, and this is the point

at which the curve increases fastest. Failures reached their highest absolute volume the second half of 1921 and the first half of 1922, but the rate of increase was less in the first half of 1921, hence the curve of cyclical differences precedes the curve of cycles. This same precedence occurs in the decline of 1922 and 1923. The first curve precedes the second curve, reaching its low point some six months ahead of the second curve.

Concerning the Regularity of the Cyclical Fluctuations.—At the beginning of the chapter it was stated that from an *a priori* point of view there seemed to be great irregularity in the cyclical movements of Business Failures. We can now make the proper modification of that statement. There is great irregularity in the directions of movement from month to month as the zigzag appearance of both curves shows, but there is also a regularity predominating the changes that have occurred. There is a complete cycle whose period may be considered as covering the years 1915 to 1922. There are parts of cycles before and after this one, the directions of which are quite evident, even in the presence of the many irregularities. Insofar as these swings of the data can be recognized, the first judgment is freely modified in accordance with the results of the analysis.

Carload Shipments of Automobiles

Solution.—The formal application of the direct method to this series presents nothing different from its application as given in Tables I, IV and VI, hence no explanation of the details of Table VIII will be necessary. The relative cyclical differences of the Table are represented by the heavy line of Chart IV and the relative cycles of the older method by the light line. The relationship of the two can be studied best from the Chart, so we turn immediately to comparison of their periods.

Comparison of Relative Cyclical Differences and Relative Cycles From Chart IV. Periods.—This chart shows even more clearly than either of the preceding ones the harmony between the movements of the two curves. The first curve has more violent fluctuations than the second, but it reflects very accurately the changes of the second curve. For both curves we can distinguish clearly four complete cycles during the

TABLE VIII

CARLOAD SHIPMENTS OF AUTOMOBILES. COMPUTATION OF RELATIVE CYCLICAL DIFFERENCES

	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925
J	11.11	9.67 87.0 18.9	21.20 219.2 188.1	23.29 109.9 81.6	11.53 49.5 −7.7	17.04 147.8 109.1	25.06 147.1 120.8	6.49 25.9 −75.7	15.36 236.7 193.8	35.42 230.6 212.0	46.47 131.2 117.0	33.82 72.8 53.3
F	12.17	11.97 98.4 43.3	23.58 197.0 169.0	22.39 95.0 65.6	12.03 53.7 −1.1	19.15 159.2 124.8	25.51 133.2 107.4	9.99 39.2 −26.8	19.64 196.6 163.0	36.14 184.0 165.8	52.22 144.5 131.9	38.35 73.4 56.2
M	15.07	17.19 114.1 75.7	29.62 172.3 150.0	29.44 99.4 77.0	16.73 56.8 17.4	23.74 141.9 114.1	29.33 123.5 101.0	16.29 55.5 15.0	27.75 170.3 146.5	45.00 162.2 147.5	54.55 121.2 109.1	51.00 93.5 80.6
A	16.22	18.91 116.6 81.7	27.69 146.4 122.6	27.70 100.0 76.2	17.80 64.3 27.3	25.27 142.0 115.9	17.15 67.9 29.5	20.19 117.7 85.0	31.33 155.2 134.2	46.10 147.1 132.8	48.06 104.3 90.6	53.72 111.8 99.5
M	12.41	15.39 124.0 81.2	25.12 163.2 137.0	26.45 105.3 80.4	17.83 67.4 30.4	24.50 137.4 110.5	21.98 89.7 59.7	18.61 84.7 49.3	33.42 179.6 159.9	45.40 135.8 121.3	35.51 78.2 59.6	49.50 139.4 126.1
J	8.62	17.08 198.1 159.5	24.56 143.8 117.0	21.52 87.6 57.0	15.87 73.7 32.2	22.20 139.9 110.2	22.52 101.4 72.1	20.27 90.0 57.5	34.23 168.9 149.6	40.29 117.7 101.3	26.05 64.7 39.4	46.00 176.6 162.3

TABLE VIII (*Continued*)

	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925
J	5.61	14.32 255.3 209.3	18.45 128.8 93.1	19.99 108.3 75.2	13.74 68.7 20.7	24.90 181.2 154.7	23.08 92.7 64.1	19.51 84.5 50.7	29.12 149.3 126.7	32.84 112.8 92.7	27.16 82.7 58.4	45.60 167.9 153.4
A	9.61	16.96 176.5 137.6	21.24 125.2 94.2	22.04 103.8 73.7	13.87 62.9 15.4	22.68 163.5 134.4	23.39 103.1 74.9	20.76 88.8 57.0	32.82 158.1 138.0	38.32 116.8 99.6	30.12 78.6 5o.7	48.60 161.4 147.8
S	12.21	18.94 155.1 120.3	22.09 116.6 86.8	20.54 93.0 60.8	10.90 53.1 −7.4	24.71 226.7 200.0	20.80 84.2 52.5	19.00 91.3 56.6	26.34 138.6 113.6	35.99 136.6 118.3	32.75 91.0 70.9	50.86 155.3 142.3
O	12.00	17.85 148.7 111.8	19.88 111.4 78.2	21.40 107.6 76.7	10.67 49.9 −11.9	29.84 279.7 257.6	17.21 57.7 19.4	17.81 103.5 66.5	27.10 152.2 127.9	42.24 155.9 140.3	34.60 81.9 62.8	57.78 167.0 155.6
N	7.71	17.14 222.3 183.8	18.17 106.0 69.7	18.94 104.2 69.2	9.25 48.8 −22.5	26.69 288.5 263.8	13.25 49.6 −.2	14.26 107.6 61.4	27.23 191.0 166.8	38.13 140.0 122.7	29.20 76.6 54.0	46.57 159.5 145.3
D	7.38	17.76 240.7 203.6	19.58 110.2 76.5	15.83 80.8 39.2	11.26 71.1 12.5	24.00 213.1 185.6	11.80 49.2 −6.7	12.31 104.3 50.7	27.24 221.3 197.1	34.98 128.4 109.6	31.85 91.1 70.4	40.58 127.4 111.2

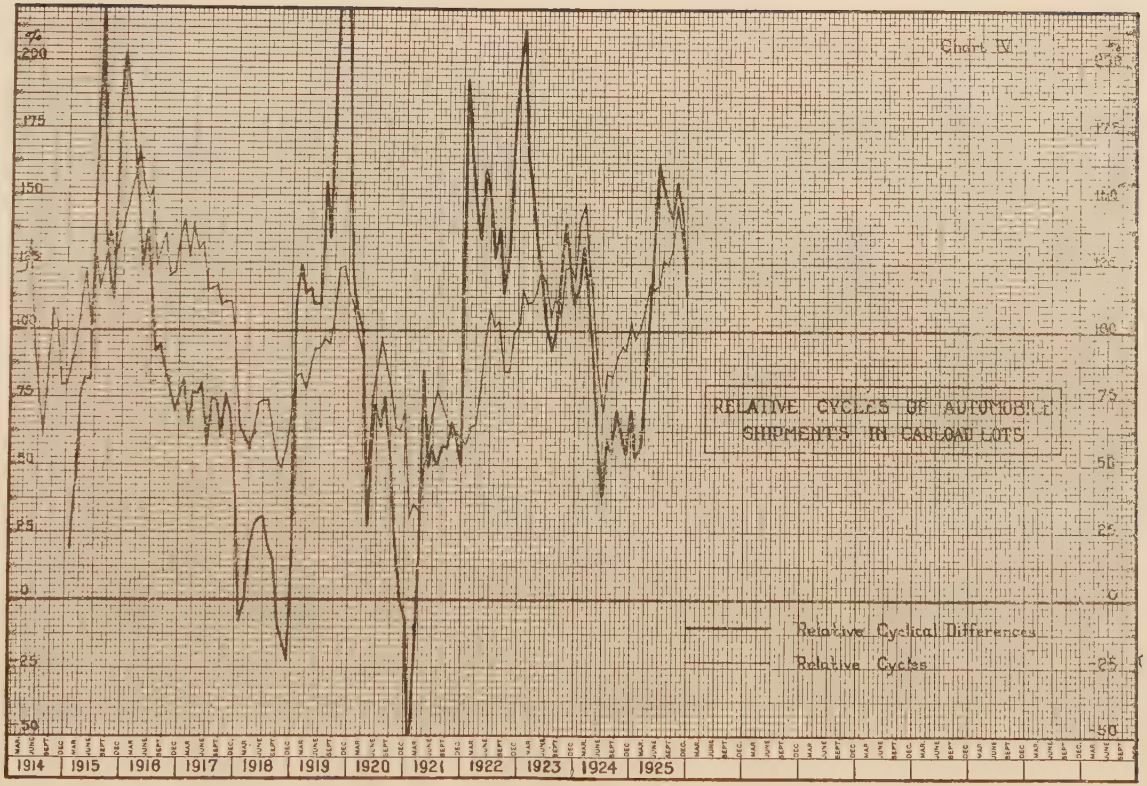


Chart IV
RELATIVE CYCLES OF AUTOMOBILE SHIPMENTS IN CARLOAD LOTS
Relative Cyclical Differences
Relative Cycles
%
200
175
150
125
100
75
50
25
0
25
50
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925

twelve-year period. We predicted differences in the periods of the cycles by the two methods, and that the curve of cyclical differences would precede in the case of a prolonged phase of the cycle. This feature is evident for the positive phase beginning with the depression of 1921. The second curve rises gradually to a high point in February 1924, whereas the first one reaches its maximum when the cyclical values are increasing fastest at the beginning of 1923. With the retarded growth near the end of 1923, the first curve declines because the cyclical differences are smaller.

There is not as much distortion during the war period of the curve representing the cycles of the older method in this series as appeared in Bradstreet's Index and Business Failures, because there was not such an increase in the volume of carload shipments of automobiles at that time. At the close of the war, when the demand for automobiles was greatly increased, the volume of carload shipments does not express the fact, because the shortage of freight cars led to a great increase in the percentage of automobiles produced which were driven from the factories to the market. This combination of events has the net effect of increasing the representativeness of straight-line trend for this series. It appears from the Chart that the curve is somewhat too high at the beginning and end of the period and too low between, but the distortion does not seem to be enough to preclude comparison of the periods of the two curves of the Chart on the basis of the points at which they are normal as well as from the standpoint of the highest and lowest points of their cyclical fluctuations. Accordingly, the dates of occurrence of the movements of the two curves are presented in Table IX.

If the results of Table IX be tabulated the periods of the cycles may be more readily recognized.

LENGTH OF CYCLE

	BY HIGHS		BY LOWS		BY NORMALS	
	NEW METHOD	OLD METHOD	NEW METHOD	OLD METHOD	NEW METHOD	OLD METHOD
1st.....	51 mo.	49 mo.		51 mo.	44 mo.	54 mo.
2nd.....	39 mo.	40 mo.	26 mo.	27 mo.	36 mo.	32 mo.
3rd.....	29 mo.	20 mo.	41 mo.	41 mo.	39 mo.	33 mo.

TABLE IX

CARLOAD SHIPMENTS OF AUTOMOBILES. DATES OF THE HIGH, LOW AND NORMAL POINTS OF THE CURVES OF CHART IV

	NEW METHOD	OLD METHOD
NORMAL		MAY, 1914
LOW		JULY, 1914
NORMAL	MAY, 1915	MARCH, 1915
HIGH	JULY, 1915	SEPTEMBER, 1915
NORMAL	JUNE, 1916	DECEMBER, 1917
LOW	NOVEMBER, 1918	OCTOBER, 1918
NORMAL	JANUARY, 1919	SEPTEMBER, 1919
HIGH	OCTOBER, 1919	OCTOBER, 1919
NORMAL	MARCH, 1920	FEBRUARY, 1920
LOW	JANUARY, 1921	JANUARY, 1921
NORMAL	JANUARY, 1922	MAY, 1922
HIGH	JANUARY, 1923	FEBRUARY, 1924
NORMAL	MARCH, 1924	APRIL, 1924
LOW	JUNE, 1924	JUNE, 1924
NORMAL	APRIL, 1925	FEBRUARY, 1925
HIGH	JUNE, 1925	OCTOBER, 1925

The evidence of this table is unmistakable. Whether measured by high points or low points, there is complete agreement of the periods of cycles by the two methods. In the case of the normal points there are disagreements arising from the distortion of the relative cycles of the older method because of the use of straight-line trend. So far, at least, as this set of data is concerned, we may conclude that not only does the new method preserve periods of cyclical fluctuations, but that it provides a more satisfactory description of the cyclical movements of the data than we obtain from the use of the older method. This last statement rests particularly on the greater elasticity and flexibility of the direct

method, and the ease with which it adapts itself to trend changes.

Amplitudes.—The direct method does not preserve amplitudes, hence no attempt will be made to interpret the amplitudes of the two curves in relation to each other. There seems to be a factor of an entirely different character operative in these data which is worthy of notice. The amplitudes of the post-war cycles are lower than the earlier ones. This phenomenon may be merely an expression of the comparatively mild character of more recent cyclical movements, but in all probability another agency is simultaneously operative. We have noted that the volume of the data has more than trebled during the period covered. With such a growth the Law of Large Numbers would indicate that the cyclical fluctuations in relative form should decrease in value. This is essentially a change in the nature of the cyclical fluctuations, hence we may conclude that, insofar as this force is present, the application of the direct method to this set of data serves to demonstrate in an actual case its adaptability to a change in the nature of the cyclical fluctuations.

Conclusions

By means of the analysis of three sets of data we have succeeded in showing the application of the method explained in Chapter II to actual cases. Each set was chosen because of a supposed peculiarity. In this way the applications were of more significance than if the data had been chosen for their regularity.

1. The results show that the direct method will lead to comparable results whether the seasonal influence be large or small; that it will eliminate a trend component which is increasing rapidly; that a rather irregular cyclical movement will appear as such.

2. Comparison of the two curves on each of the charts demonstrates their similarity of movement.

3. The general tendency of the cyclical differences to precede the cycles in their various swings is bound up with the question of rate of change of the cyclical values. On the face

of it there does not seem to be much possibility of setting up a rule for this. An individual study of each set of data from cycle to cycle must be made.

4. From the results of applying the method to the controlled data of Chapter II, we arrived at the conclusion that the precedence would be greater for long cycles than for short ones. This is borne out in each case by the cycle of 1918, 1919, 1920. No further attempts at interpretation of this phenomenon seem to be necessary.

V

Conclusions

Restatement of the Direct Method.—The problem proposed for solution in the beginning was the measurement of cycles without being forced to measure and remove trend and seasonal movements as preliminary steps. Such preliminary steps introduce unnecessary possibilities of error if the primary object is to describe cyclical changes. The method developed arrives at the relatives of cyclical differences in two major operations, first, the division of each item of data by the item of the same season of the preceding year, and secondly, the correction for trend residue. The first step removes the seasonal variations, reduces the data to relative form and, since it is a chain index, removes the normal growth. The second step removes the small residual trend element which represents the increase of one year. Each result is the difference between two cyclical values occurring one year apart, expressed as a relative of the original data of the earlier year.

Comparison of Assumptions of the Direct Method With Those Used in Older Methods.—Neither trend nor seasonal variation are specifically measured at any time. For this reason it is possible to make two rather flexible assumptions concerning them, and pass directly to the cycle. The two assumptions are:

1. The amount of a time series which is to be attributed to seasonal variation will not be materially different for the same seasons of any two adjoining years.

2. Secular trend is considered to be a slightly waving curve fitting a set of data and responding remotely to the short-time oscillations thereof.

These two assumptions are used to replace the ones used in

older methods, which may be restated for comparative purposes:

1. Seasonal variation repeats itself with equal relative intensity from year to year.

2. Secular trend may be represented by a straight line fitted to the data.

The first two are decidedly elastic as compared with the older set. The first is flexible enough to allow the seasonal variation to change in value over a period of years, and the second is flexible enough to allow for changes in the direction of the trend through the years.

In the final result we have not measured cycles but cyclical differences. It is this introduction of a new concept which has required the greatest amount of explanation. We have demonstrated that the relative cyclical differences lead to a wave curve whose periodic changes conform to those of a curve of relative cycles. Further, that the amplitudes of the curve of cyclical differences express rapidity of change whereas the amplitudes of the curve of relative cycles express amounts which are the results of such change. The cycle with the largest volume of data will have the highest amplitude by the older methods, the cycle with the greatest volume of change will have the highest amplitude by the new method.

Final Test of Experience.—The final test of whether this is a feasible method of representing cyclical movements must come from an appeal to business conditions. In applying it to the controlled data in which the amount to be attributed to cycle was known in advance it was not found wanting. Further, the test of actual data was successfully met in that this method expressed the cycle equally as well as older methods did. As a matter of fact it excelled older methods because of the facility with which it met and interpreted the difficulties of the data chosen arising from the particular period taken. And it has the added advantage of ease of computation and the elimination of the intermediate steps of specifically removing trend and seasonal variation. But in any set of actual data the exact amount due to cyclical change cannot be known, so that this method along

with all others must meet a last test. Does it adequately express the cyclical movements of the data to which it is applied? The writer feels that the answer to this question cannot result from the application of any formal process, but is in the last analysis based purely on individual judgment. And it appears that the method presented here has satisfactorily fulfilled every reasonable requirement for the cases to which it has been applied.